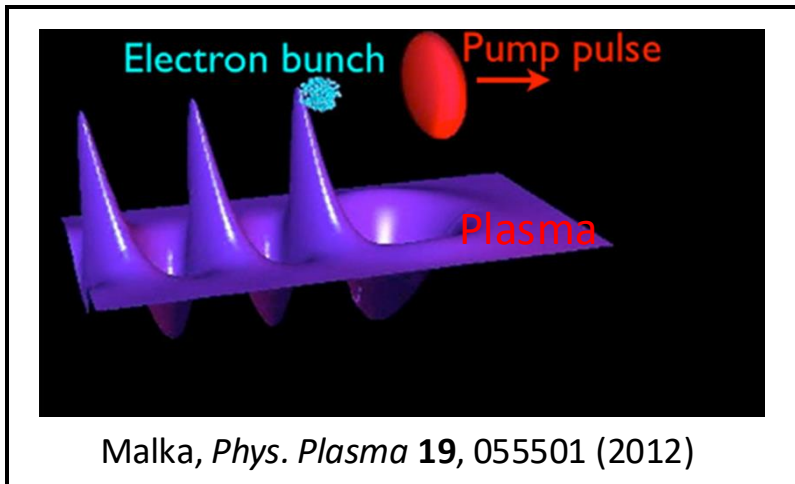
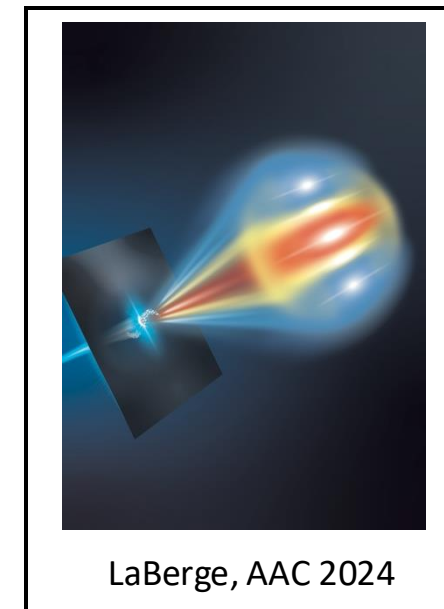


# Exploring the quasi-6D structure of laser-wakefield-accelerated electron bunches with coherent optical transition radiation



Ze Ouyang  
Supervisor: Michael Downer  
6<sup>th</sup> Nov, 2024



# Outline

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1

Introduction to LWFA and its diagnostics

2

COTR(I) and quasi-6D structure of e- bunches

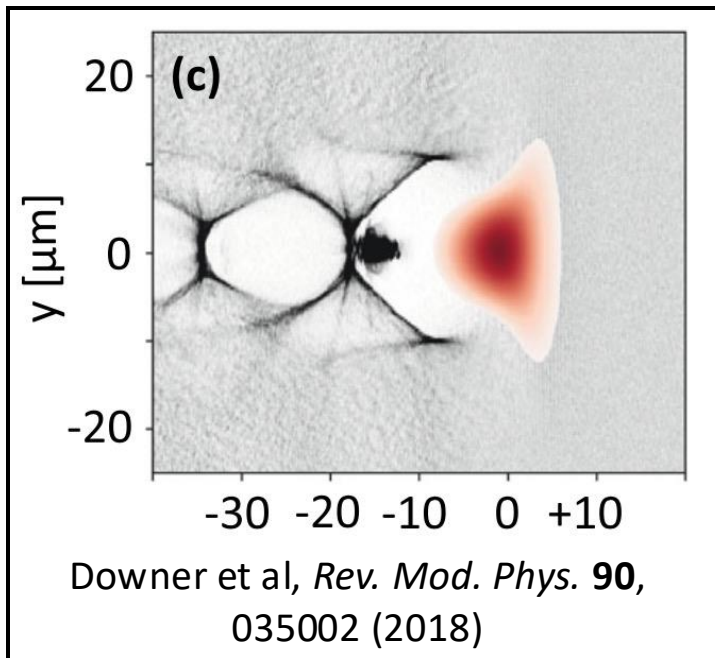
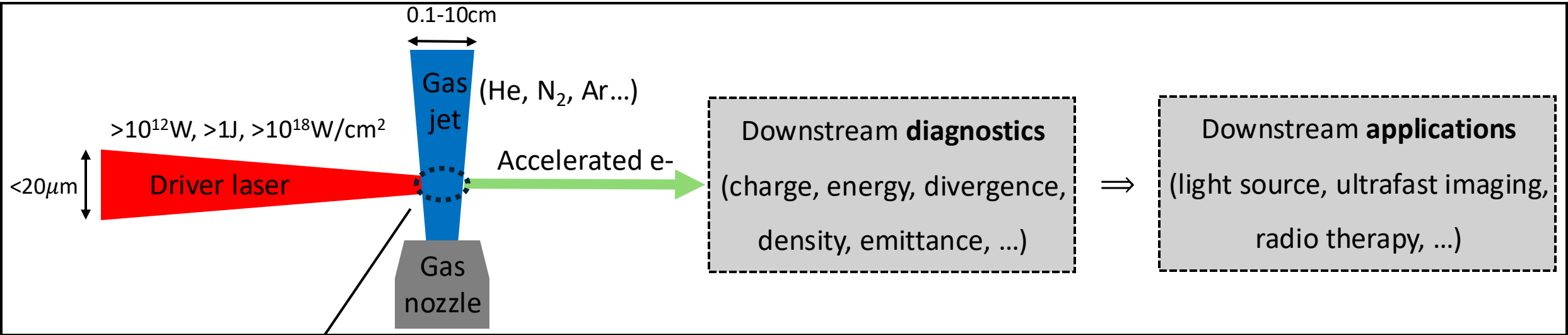
3

Future directions, experimental work & conclusion

Useful abbreviations:

- LWFA: Laser-driven WakeField Accelerator
- TR: Transition Radiation
- COTR: Coherent Optical Transition Radiation
- COTRI: Coherent Optical Transition Radiation Interferometry

# Introduction: Laser-driven WakeField Accelerator



LWFA ∈ plasma-based accelerator<sup>1</sup>

	Plasma-	Conventional (SLAC)
$E$	100GV/m	100MV/m
Footprint	~m	~km
Max Energy	10GeV <sup>2</sup>	50GeV
Cost	~few \$millions	114 \$millions in 1960s

Plasma-	Conventional
<100pC	~nC
~μm	~10μm
<10fs	~100fs
~mrad	~μrad

We need new diagnostics.

<sup>1</sup> Tajima et al, *Phys. Rev. Lett.* **43**, 4 (1979)

<sup>2</sup> Aniculaesei et al, *MRE* **9**, 014001 (2024)

# Introduction: LWFA diagnostics

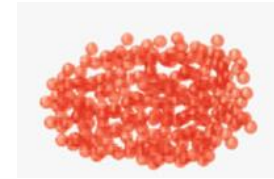
e- beams from LWFA can be:

- transversely small:  $0.1 \mu\text{m} < \sigma_r < 1 \mu\text{m}$
- longitudinally short:  $0.03 \mu\text{m} < \sigma_z < 3 \mu\text{m}$  ( $0.1 \text{ fs} < \sigma_z/c < 10 \text{ fs}$ )
- highly divergent:  $1 \text{ mrad} < \sigma_r' < 10 \text{ mrad}$   
 $\Rightarrow$  transverse normalized emittance:  $0.1 \text{ mm mrad} < \varepsilon_n < 1 \text{ mm mrad}$
- **microbunched**: e- grouped into subtle structure within sub- $\mu\text{m}$  range  
**(Today's diagnostics frontier)**
- bunch charge, energy spread, repetition rate, efficiency et al.

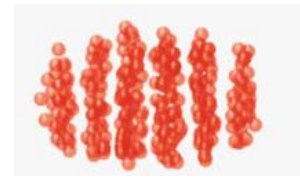
$$\text{Emittance}^1: \varepsilon_x \equiv \sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle xx' \rangle^2}$$

$$\text{Normalized emittance: } \varepsilon_{x,n} = \beta_z \gamma \varepsilon_x \approx \gamma \varepsilon_x$$

1.  $\propto$  area of e- occupied in 6D phase space
2. Conserved in ideal beam transportation



non-mb



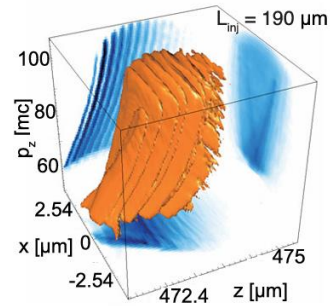
mb

*Eur. Phys. J. Spec. Top.* 233: 1-208 (2024)

- Microbunched e- structure (**only**) by COTR (3D)
- Transverse divergence by COTRI (2D)
- z-dependent transverse divergence by COTRI and physical constraints (quasi-1D)

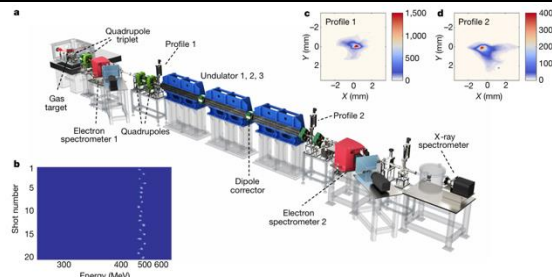
COTR  $\Rightarrow$  **quasi-6D structure**

1 Corde et al, *Rev. Mod. Phys.* **85**, 000001 (2013)



“Microbunched e- beam in LWFA”

Xu et al, *Phys. Rev. Lett* **117**, 034801 (2016)



“Microbunched e- beam in LWFA-based Free Electron Laser”

Wang et al. *Nature* **595**, 516-520 (2021)

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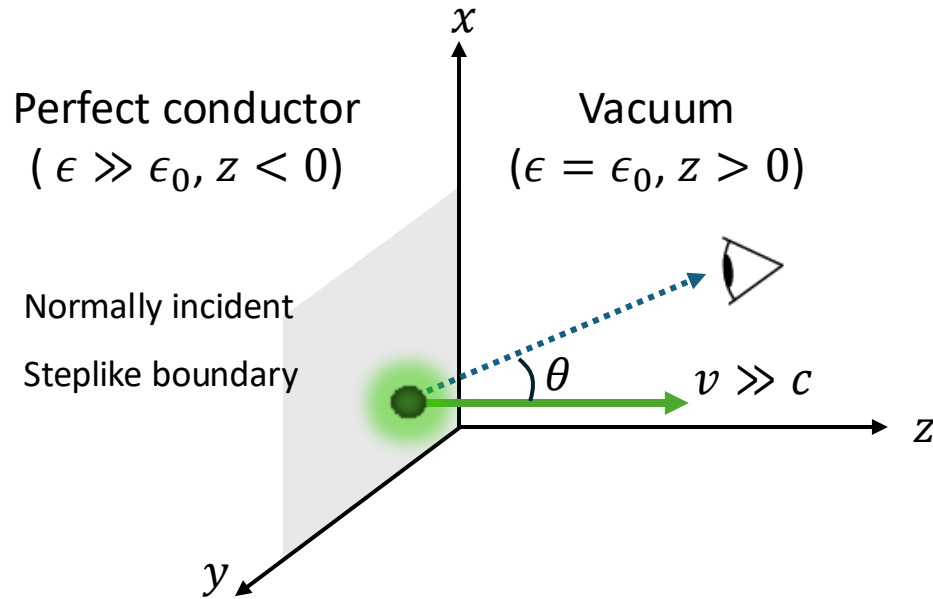
Future directions, experimental work & conclusion

Useful abbreviations:

- LWFA: Laser-driven WakeField Accelerator
- TR: Transition Radiation
- COTR: Coherent Optical Transition Radiation
- COTRI: Coherent Optical Transition Radiation Interferometry

# Transition Radiation (single e-)

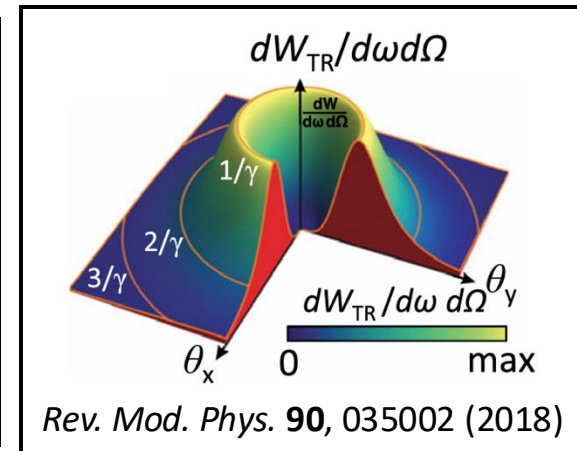
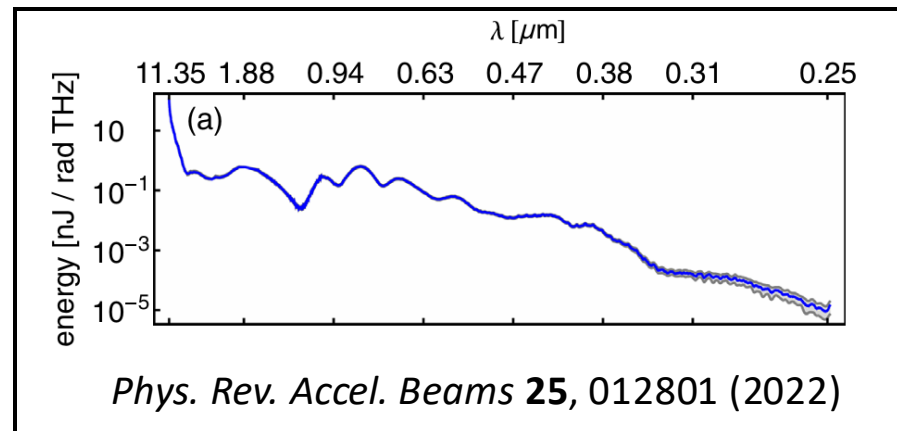
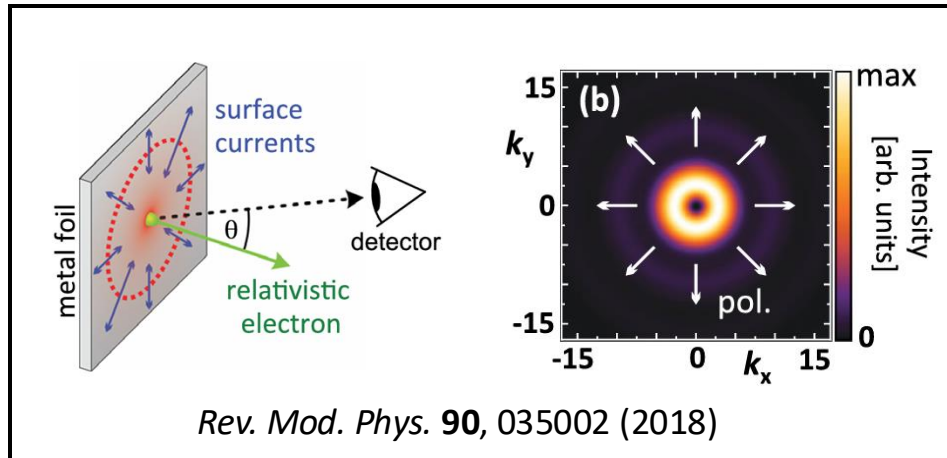
TR is emitted when charged particle passes from one medium into another with different index of refractive.



Single e- TR energy<sup>1</sup> in far field :

$$\frac{d^2 W_1}{d\omega d\Omega} = \frac{e^2}{4\pi^3 \epsilon_0 c} \frac{\beta^2 \sin^2 \theta}{(1 - \beta^2 \cos^2 \theta)^2}$$

1. target radiating & radially polarized
2. broadband (low- and high-  $\omega$  cutoff:  $0.2\mu\text{m}$ - $10\mu\text{m}$ )
3. narrow cone (peaked at  $\theta \sim \frac{1}{\gamma}$ ) & weakly  $\gamma$ -dependent ( $\gamma \gg 1$ )

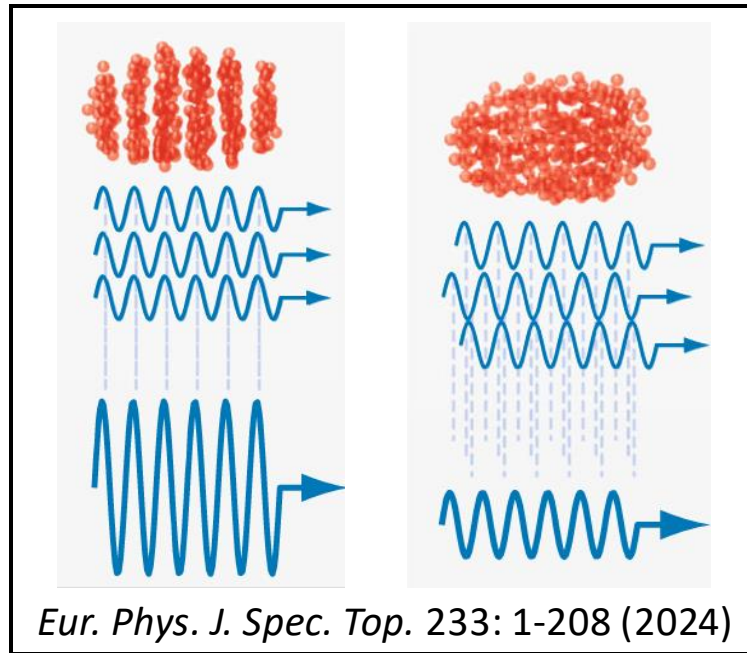


# Transition Radiation (e- bunch)

In the case of multiple e-:

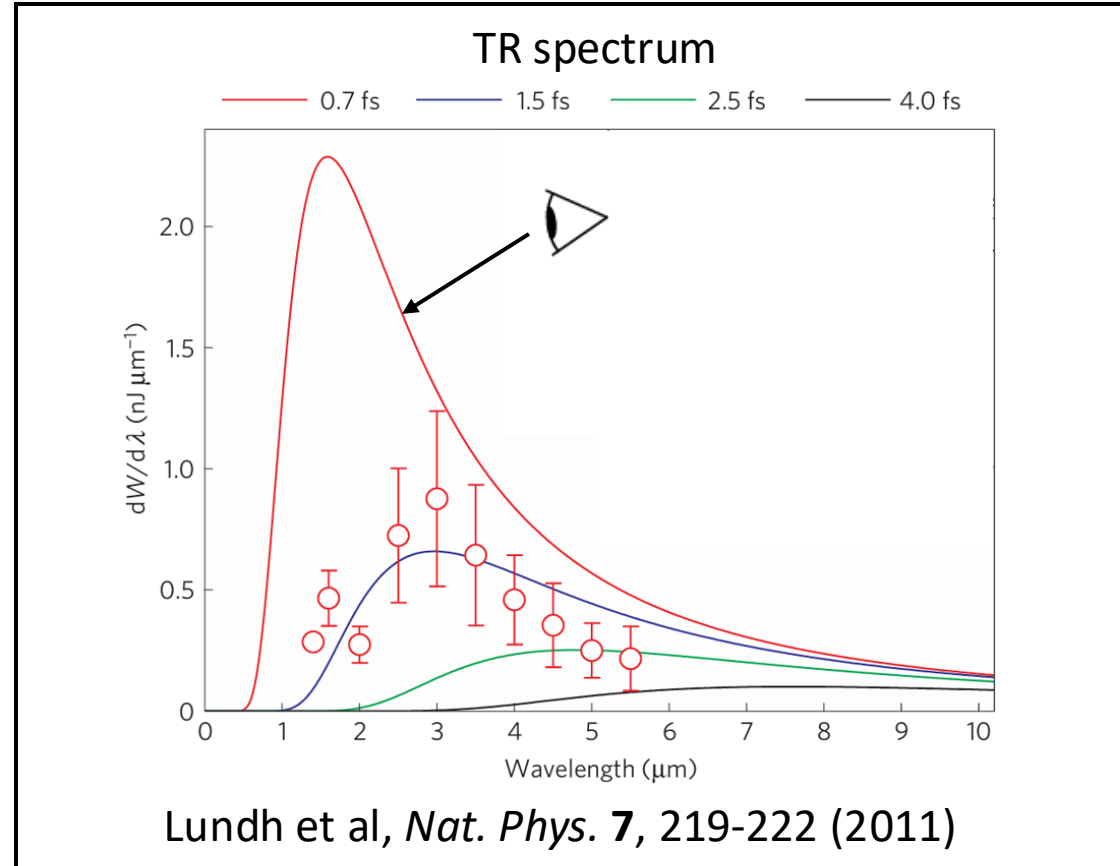
$$\frac{d^2 W_N}{d\omega d\Omega} = [N + N(N-1) \cdot |F(\omega, \theta)|^2] \cdot \frac{d^2 W_1}{d\omega d\Omega}$$

- Out-of-phase emission  $\propto N$  (incoherent)
- In-phase emission  $\propto N^2$  (coherent)



where  $F(\omega, \theta)$  is the **form factor** (level of coherence)

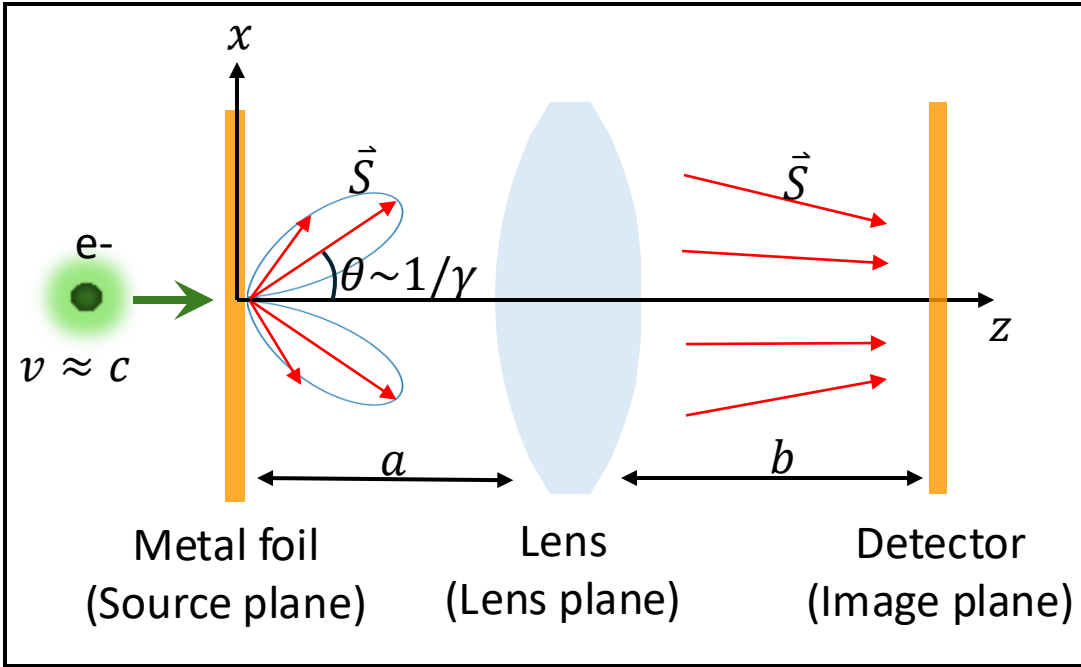
$$F(\omega, \theta) = \int \rho(\mathbf{r}) e^{i\mathbf{k}\mathbf{r}} d\mathbf{r}$$



- $\lambda > \sigma_z$ : incoherent
- $\lambda < \sigma_z$ : coherent
- microbunched e- beam:  $\lambda$  is coherent down to **optical range (COTR)**  $\Rightarrow$  structure info

# Transition Radiation Imaging (single e- near field)

COTR is detected in the near field



$$\left. \begin{array}{l} \lambda \\ \gamma \\ M \\ \theta_m \end{array} \right\} \Rightarrow S(x_i, y_i, \omega)$$

Source plane<sup>1,2</sup>:

$$E_{x,y}^s(x_s, y_s, \omega) = \frac{e\omega}{\pi v^2 \gamma} \frac{x_s, y_s}{\sqrt{x_s^2 + y_s^2}} K_1 \left( \frac{\omega}{v\gamma} \sqrt{x_s^2 + y_s^2} \right)$$

Lens plane:

$$E_{x,y}^{li}(x_s, y_s, \omega) = -\frac{ie^{ika}}{\lambda a} e^{ik\frac{x_l^2 + y_l^2}{2a}} \int dx_s dy_s E_{x,y}^s e^{-ik\frac{x_l x_s + y_l y_s}{2a}} e^{ik\frac{x_s^2 + y_s^2}{2a}}$$

$$E_{x,y}^{lo}(x_s, y_s, \omega) = E_{x,y}^{li}(x_s, y_s, \omega) e^{-ik\frac{x_l^2 + y_l^2}{2f}}$$

Image plane:

$$E_{x,y}^i(x_s, y_s, \omega) = -\frac{ie^{ikb}}{\lambda b} e^{ik\frac{x_i^2 + y_i^2}{2b}} \int dx_l dy_l E_{x,y}^{lo} e^{-ik\frac{x_l x_i + y_l y_i}{2b}} e^{ik\frac{x_l^2 + y_l^2}{2b}}$$

$$\Rightarrow \mathbf{E}(x_i, y_i) = \frac{2e}{\lambda v M} f(\theta_m, \gamma, \zeta) \mathbf{e}_r \text{ Field Point Spread Function (FPSF)}$$

The energy flux per unit frequency interval is

$$S(x_i, y_i, \omega) = \frac{c}{4\pi^2} (|\mathbf{E}(x_i, y_i)|^2) = \frac{d^3 W_1}{d\omega dx_i dy_i} \text{ Point Spread Function (PSF)}$$

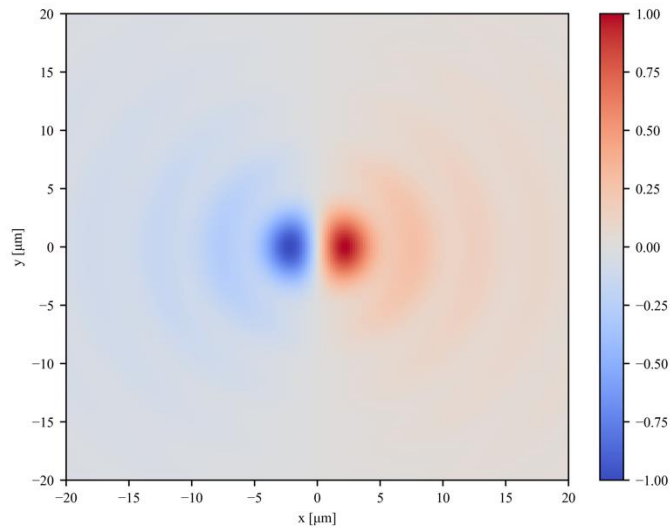
<sup>1</sup> Castellano et al. *Phys. Rev. Accel. Beams* **1**, 062801 (1998)

<sup>2</sup> Xiang et al. *Nucl. Instrum. Meth. A* **570**, 3 (2007)

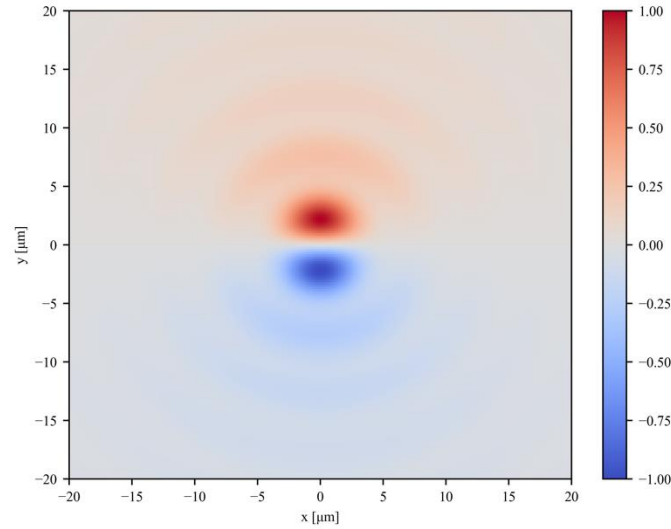


# Transition Radiation Imaging (single e- near field)

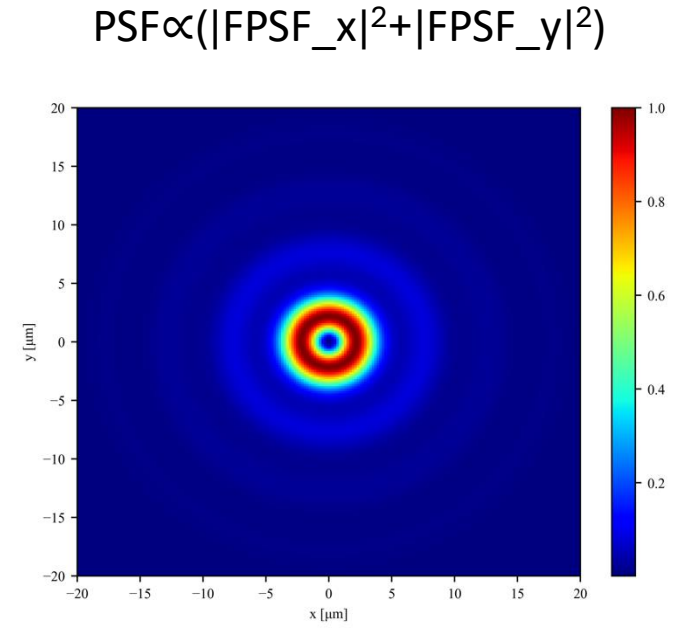
$\lambda=500\text{nm}$ ,  $M=1$ ,  $\gamma=391$  (200MeV), and  $\theta_m=0.1$



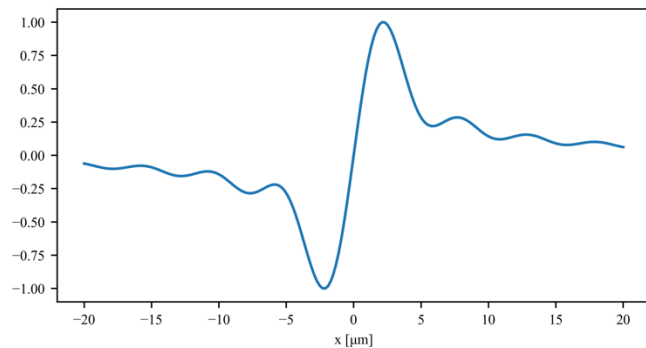
FPSF polarized in x-axis



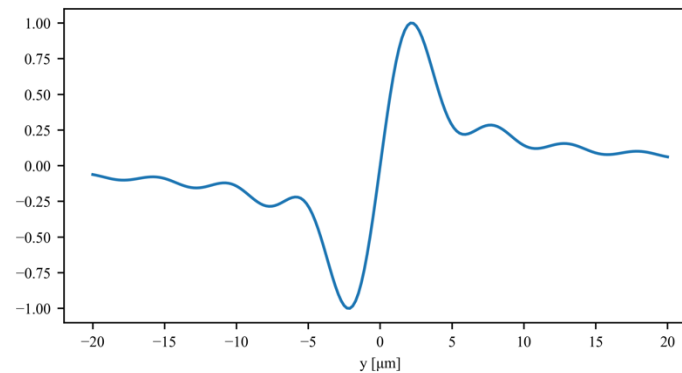
FPSF polarized in y-axis



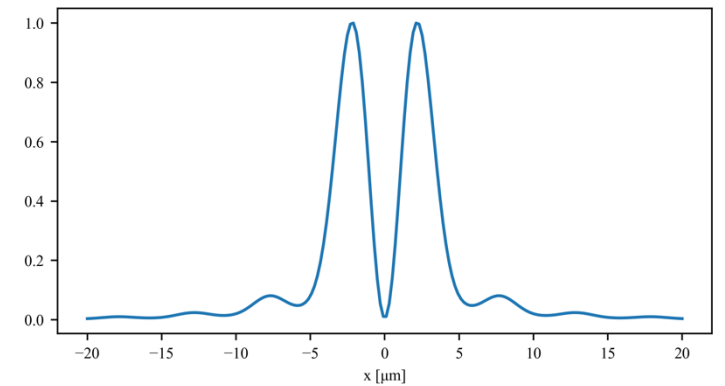
PSF



Lineout of FPSF\_x at y=0

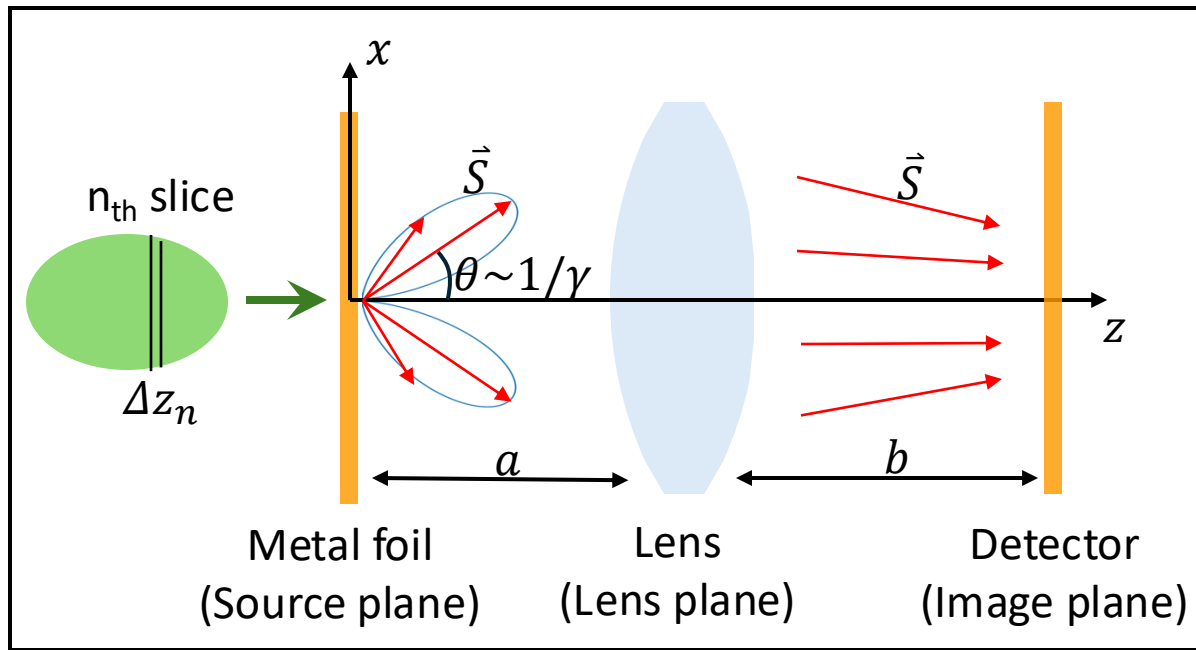


Lineout of FPSF\_y at x=0



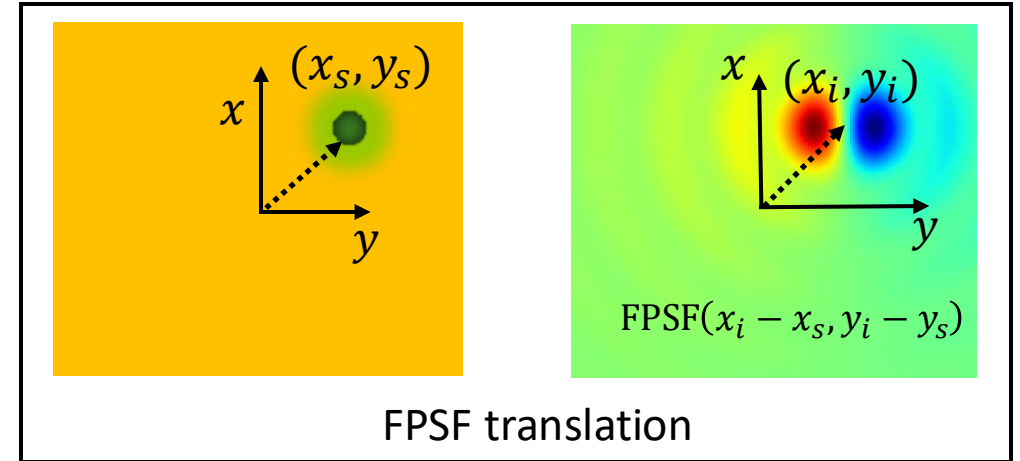
Lineout of PSF at y=0

# Transition Radiation Imaging (e- bunch near field)



Electron number density  $\rho(x_s, y_s, z_s)$

$$E^{(n)}(x_i, y_i) = \Delta z_n \iint dx_s dy_s \rho(x_s, y_s, z_n) \text{FPSF}(x_i - x_s, y_i - y_s)$$



Each slice has a phase delay<sup>1</sup>  $e^{ikz_n}$

Total  $\mathbf{E}$  field is

$$\mathbf{E}_{\text{tot}}(x_i, y_i) = \iiint dx_s dy_s dz_s \cdot \rho(x_s, y_s, z_s) \cdot e^{ikz} \cdot \text{FPSF}(x_i - x_s, y_i - y_s)$$

Total energy flux per unit frequency interval is

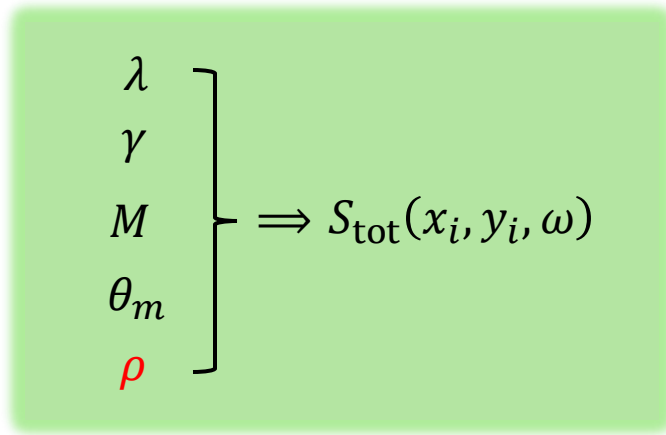
$$S_{\text{tot}}(x_i, y_i, \omega) = \frac{c}{4\pi^2} (|\mathbf{E}_{\text{tot}}(x_i, y_i)|^2) = \frac{d^3 W_1}{d\omega dx_i dy_i}$$

$$\left. \begin{array}{l} \lambda \\ \gamma \\ M \\ \theta_m \\ \rho \end{array} \right\} \Rightarrow S_{\text{tot}}(x_i, y_i, \omega)$$

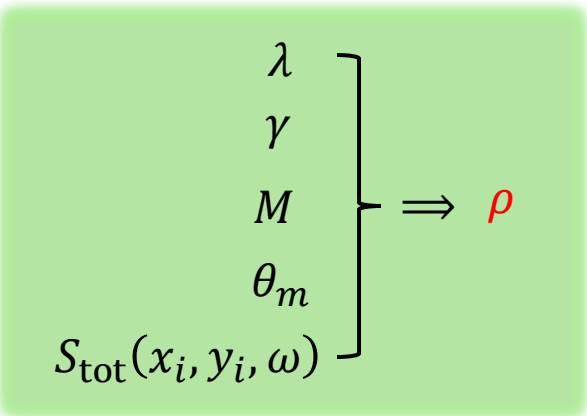
**S contains info of  $\rho$**

# Revealing the $\rho(x_s, y_s, z_s)$ by COTR: an inverse problem

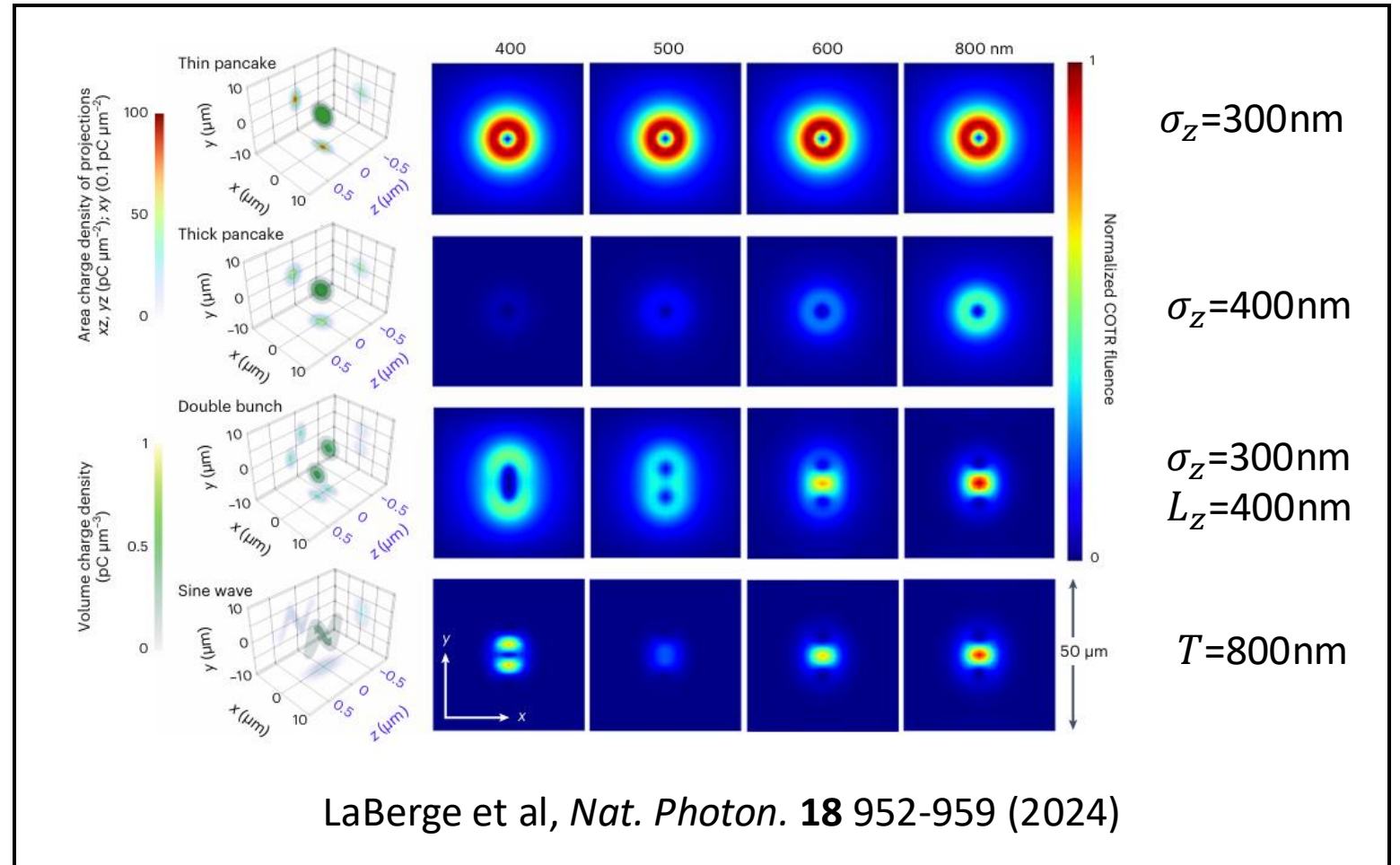
Forward process:  $\rho(x_s, y_s, z_s) \Rightarrow S(x_i, y_i)$



Backward process:  $S(x_i, y_i) \Rightarrow \rho(x_s, y_s, z_s)$



Without loss of generality, consider  $S$  as what is measured.

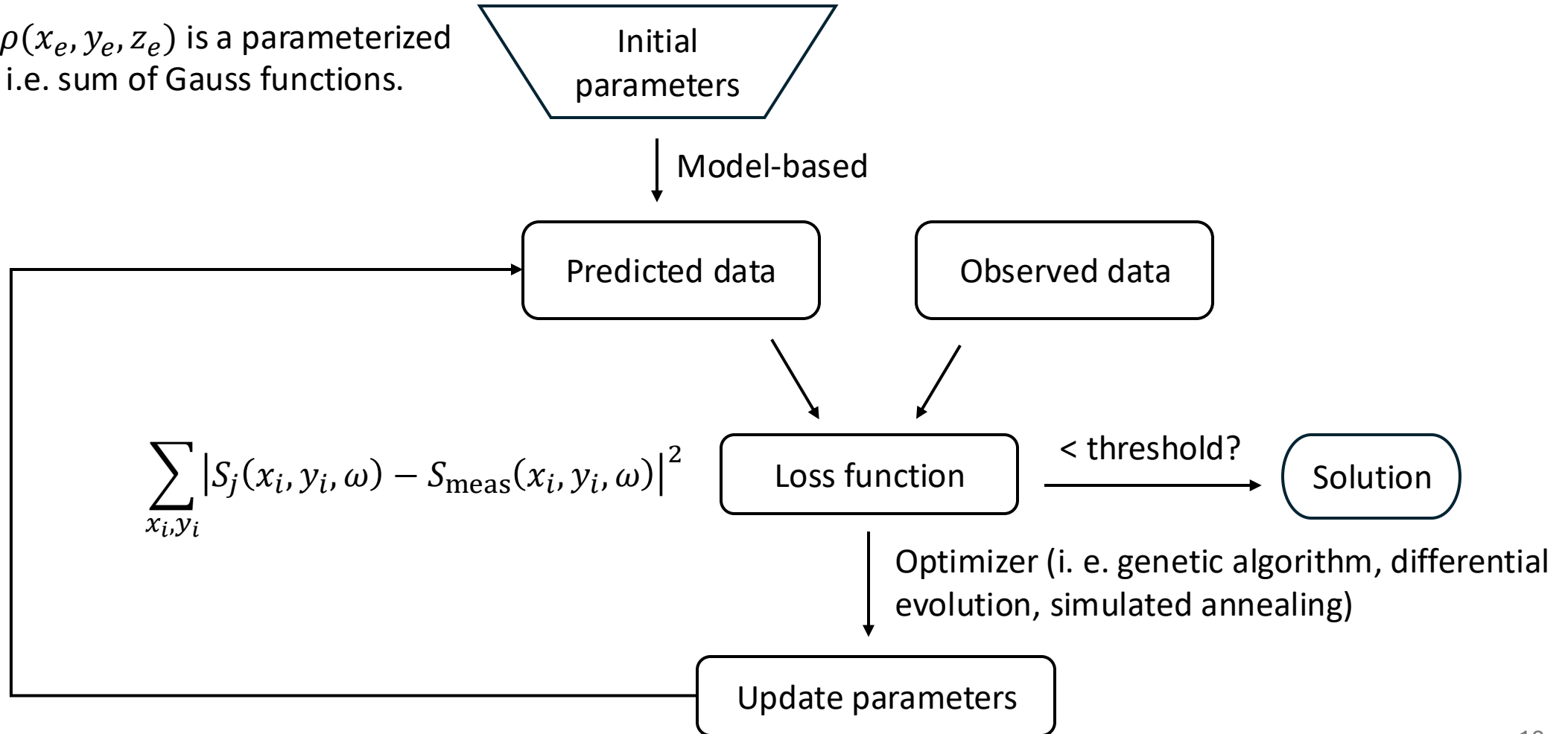


An inverse problem

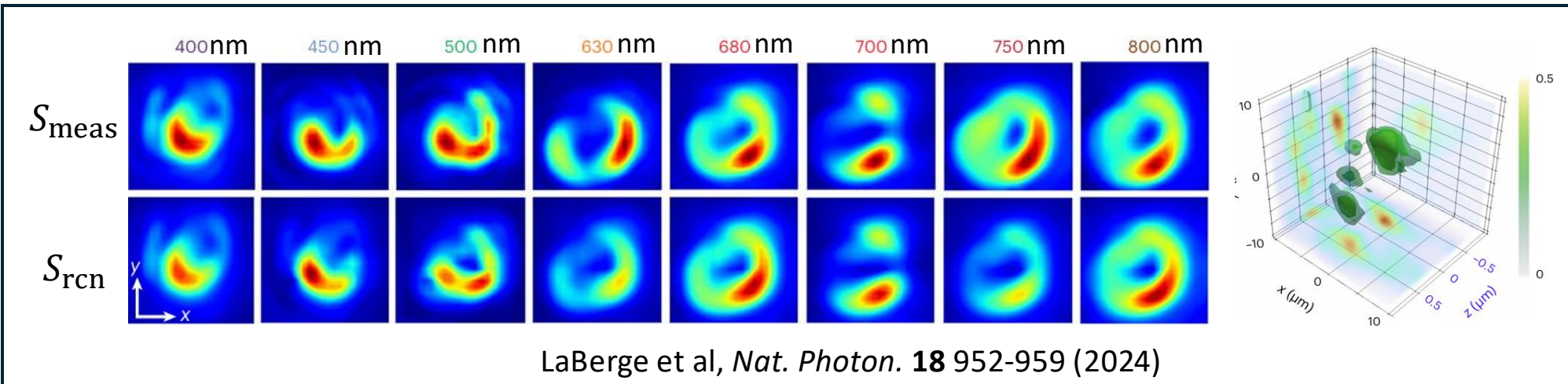
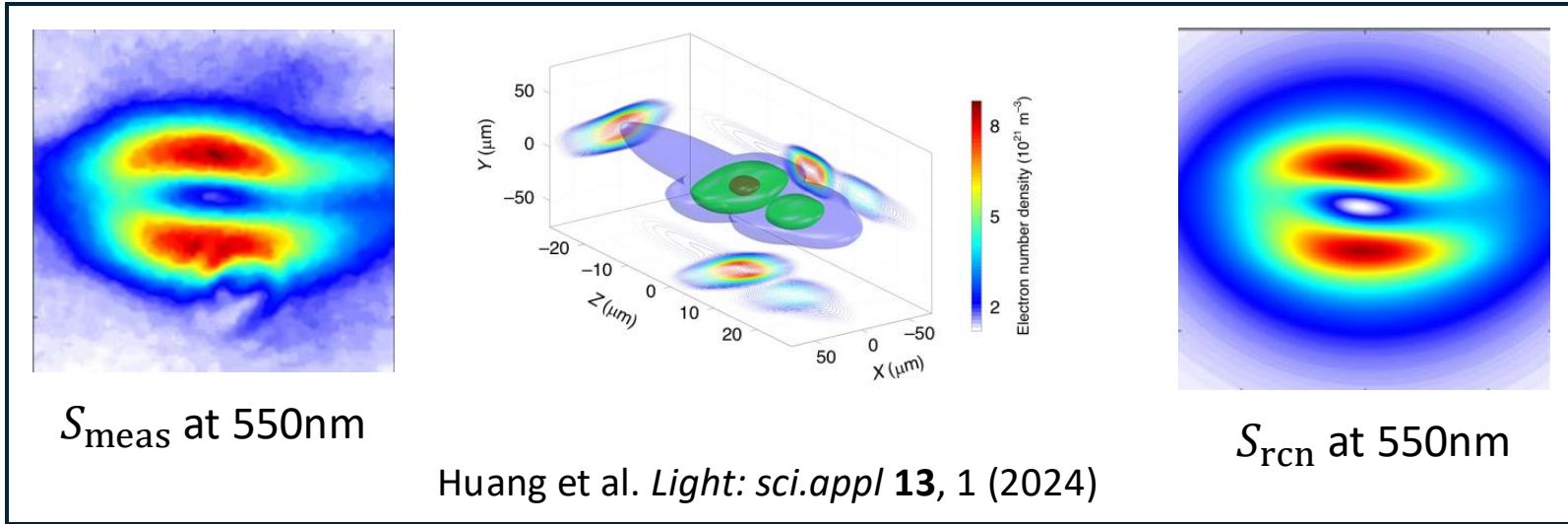
# Revealing the $\rho(x_s, y_s, z_s)$ by COTR: workflow

$$\rho(x_s, y_s, z_s) = \sum_{j=1}^N N_{e_j} \frac{1}{\sqrt{2\pi}\sigma_{x_j}} \exp\left(-\frac{(x-\mu_{x_j})^2}{2\sigma_{x_j}^2}\right) \frac{1}{\sqrt{2\pi}\sigma_{y_j}} \exp\left(-\frac{(y-\mu_{y_j})^2}{2\sigma_{y_j}^2}\right) \frac{1}{\sqrt{2\pi}\sigma_{z_j}} \exp\left(-\frac{(z-\mu_{z_j})^2}{2\sigma_{z_j}^2}\right)$$

Suppose  $\rho(x_e, y_e, z_e)$  is a parameterized function, i.e. sum of Gauss functions.



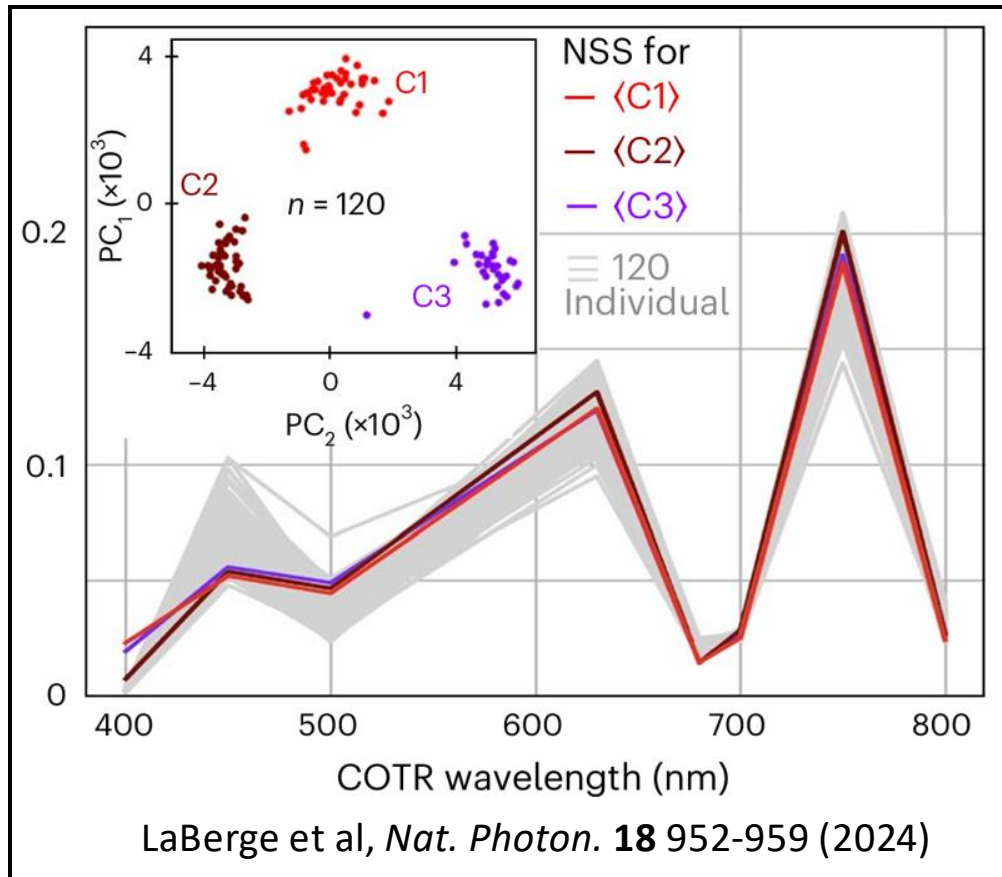
# Revealing the $\rho(x_s, y_s, z_s)$ by COTR: Latest results



# Revealing the $\rho(x_s, y_s, z_s)$ by COTR: uniqueness

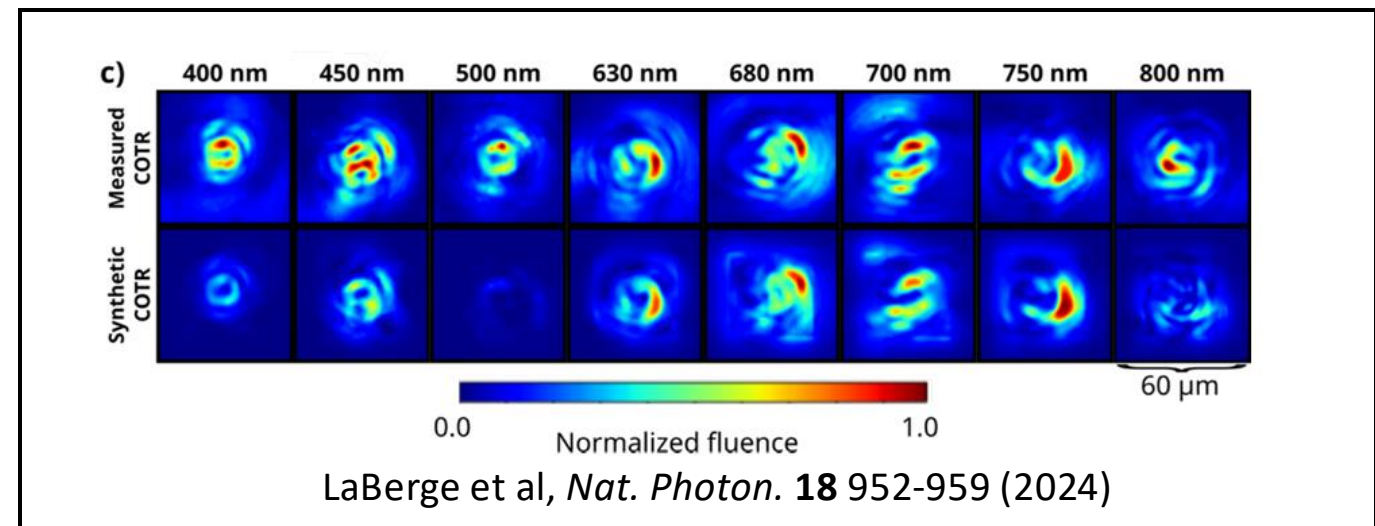
Phase info lost in the forward process  $\Rightarrow$  reconstruction is not unique

How to compress the volume of solution space  $\Rightarrow$  **Knowing longitudinal profile in advance!**



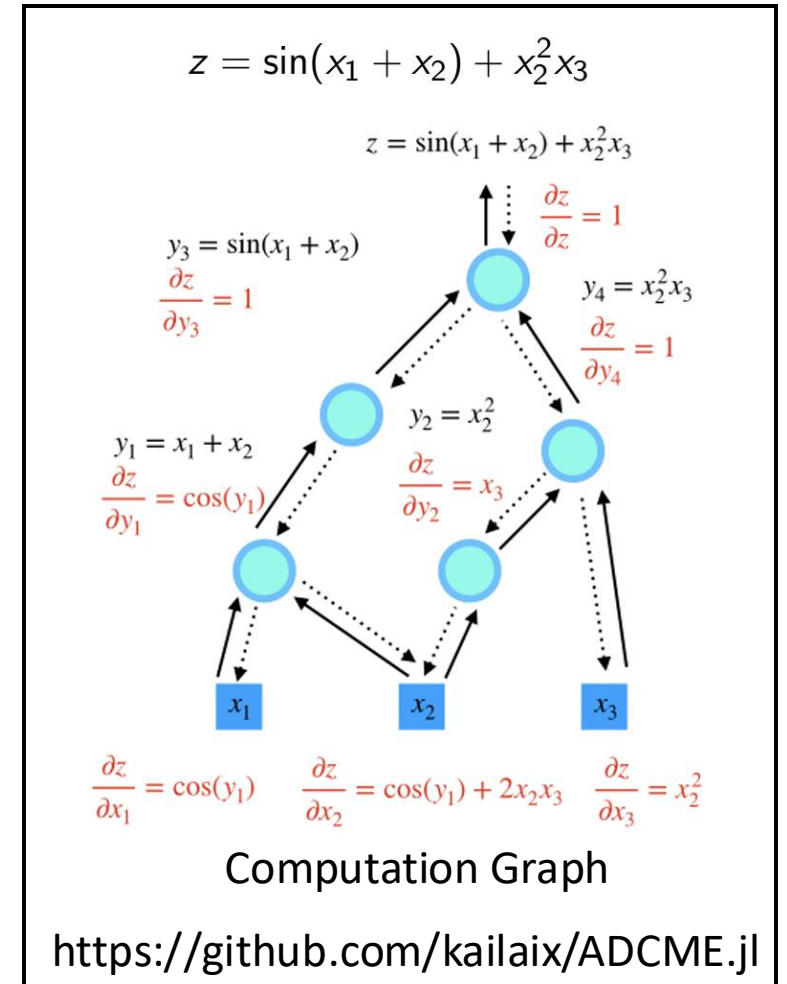
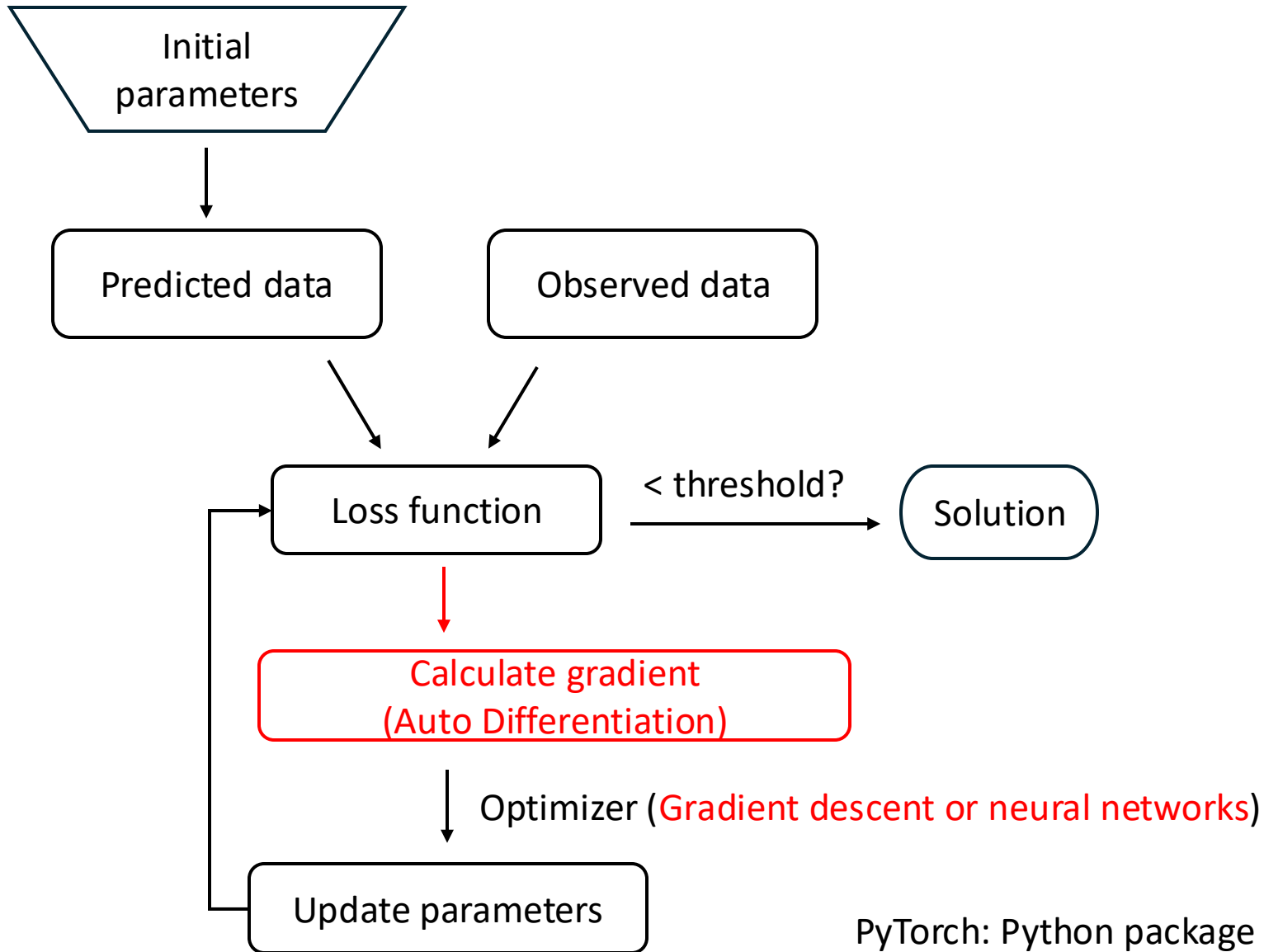
Knowledge of e- beam longitudinal is **injection-regime-dependent:**

- Down ramp injection: e- spectrum
- Self-truncated ionization injection: PIC simulation
- Self injection: not accessible



**Other architectures to solve such an inverse question?**

# Revealing the $\rho(x_s, y_s, z_s)$ by COTR: ML-workflow

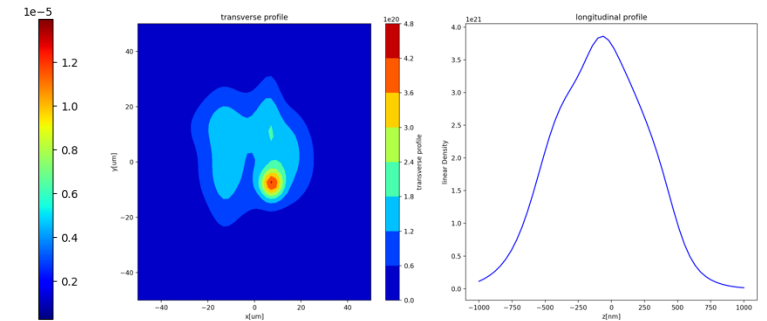
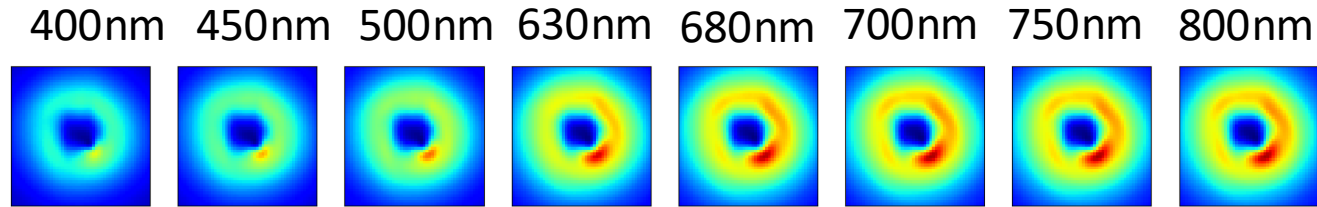


PyTorch: Python package for

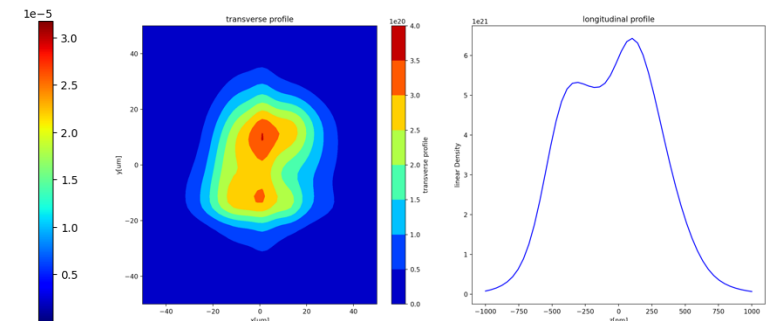
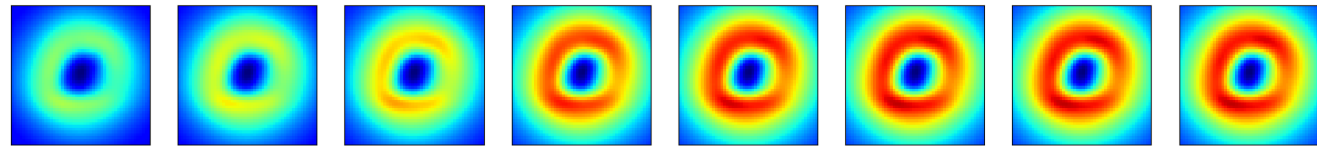
- GPU computation
- Auto Differentiation (AD)
- Machine Learning (ML)

# Revealing the $\rho(x_s, y_s, z_s)$ by COTR: Gradient descent

“ $S_{\text{meas}}$ ”  
26 Gauss

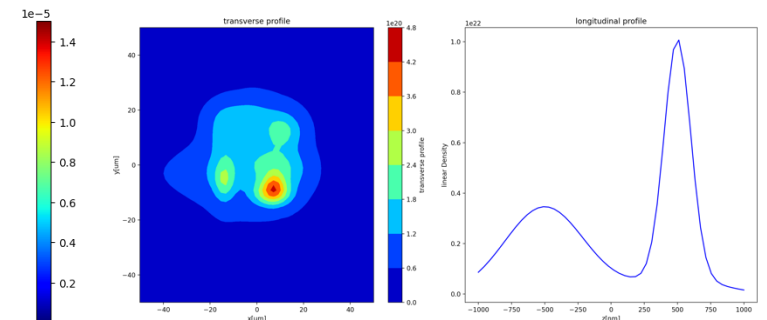
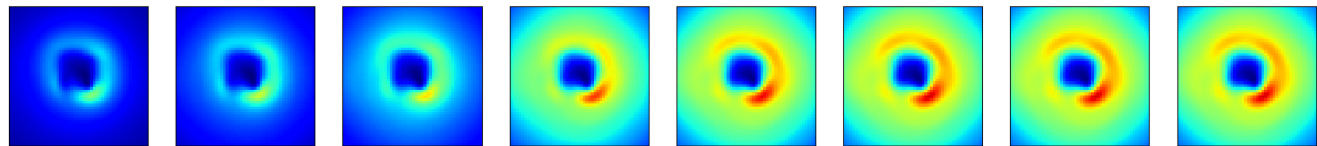


$S_{\text{seed}}$   
50 Gauss



Loss function:  $\sum_{x_i, y_i} |S_j(x_i, y_i, \omega) - S_{\text{meas}}(x_i, y_i, \omega)|^2$  iterate

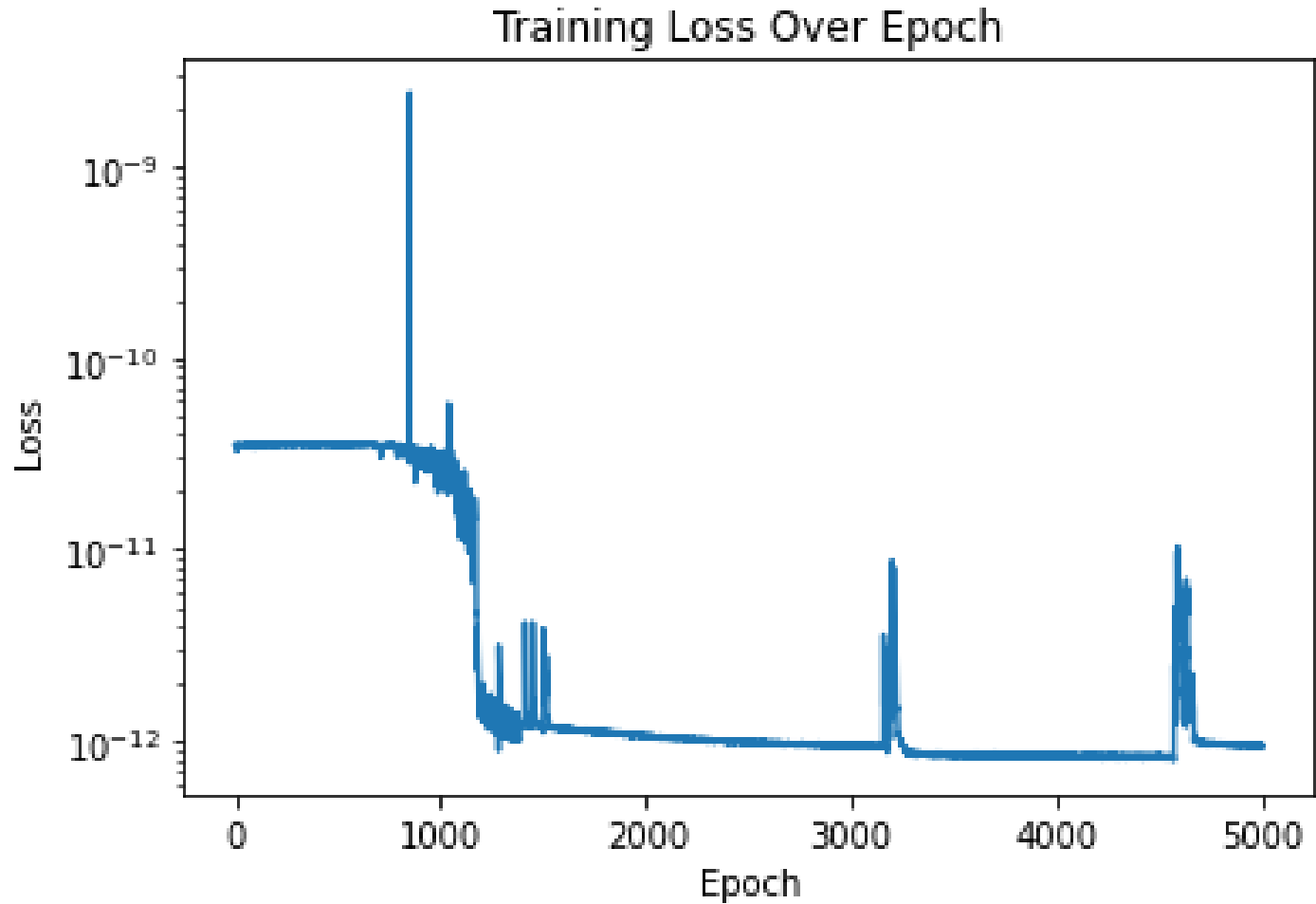
$S_{\text{rcn}}$   
50 Gauss





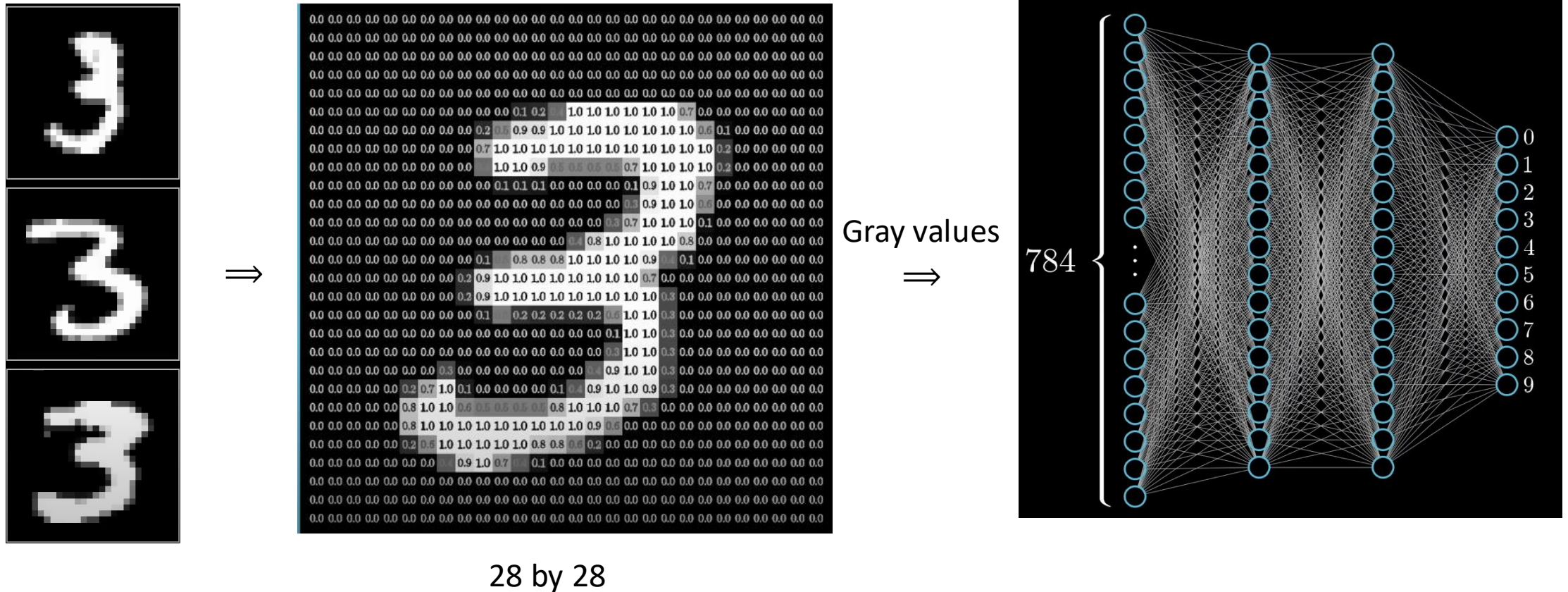
# Revealing the $\rho(x_s, y_s, z_s)$ by COTR: Training loss

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- ~2 hours
- Final loss reduced to 1/50 of the initial loss

# Revealing the $\rho(x_s, y_s, z_s)$ by COTR: Neural network “vision”

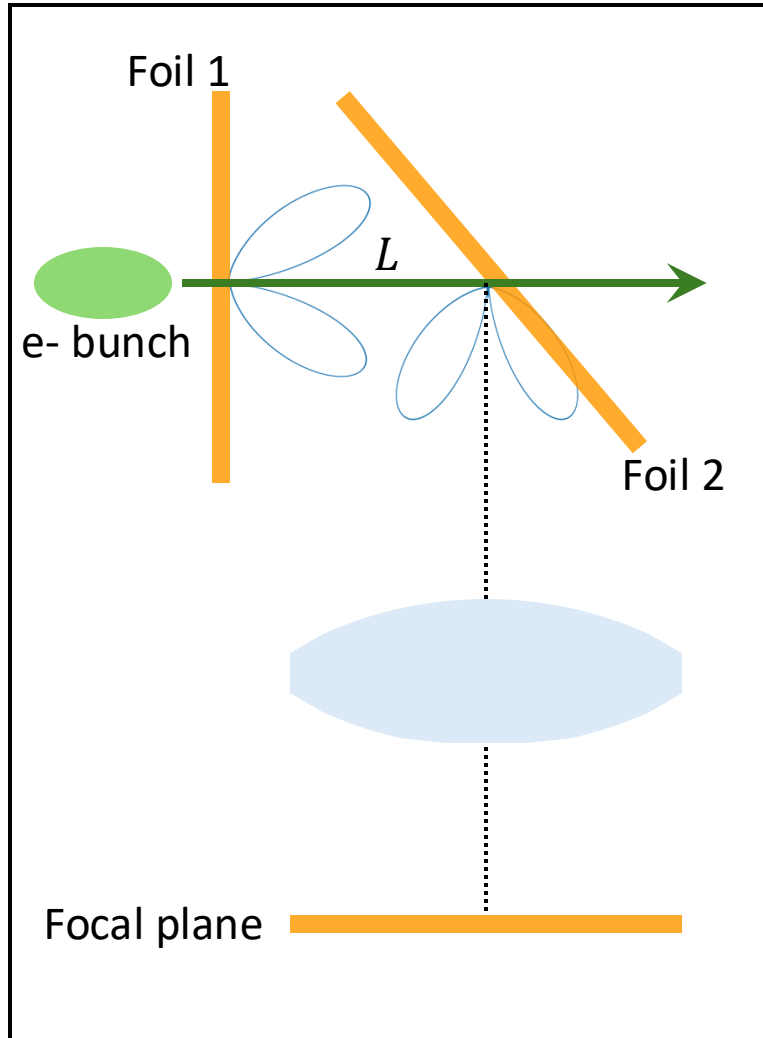


Training repository: paired  $\rho$  and  $S$  for NN(neural network) to learn

Test repository: paired  $\rho$  and  $S$ . Given the  $S$ , to see if the NN could deduce  $\rho$  close to the right one

# COTRI Imaging

COTRI is detected in the far field

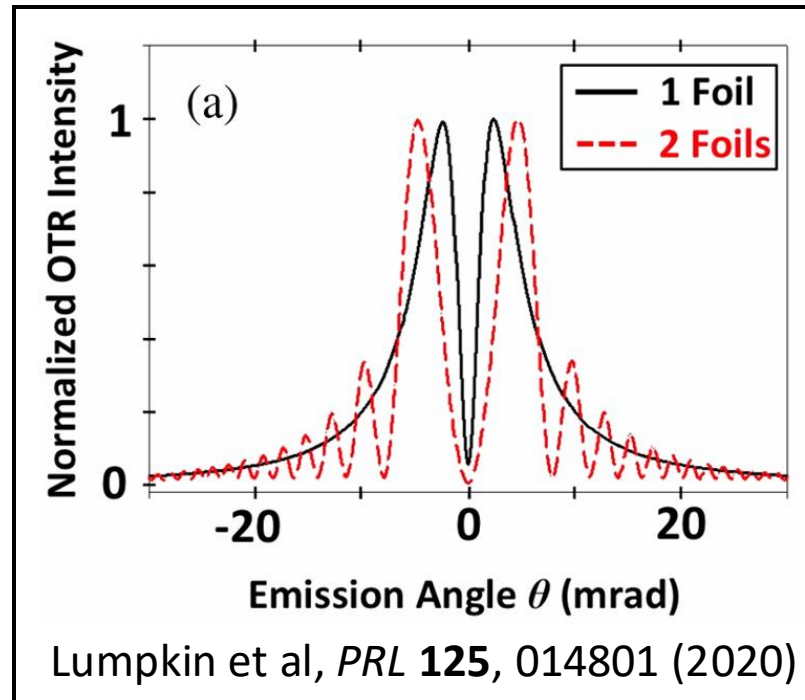
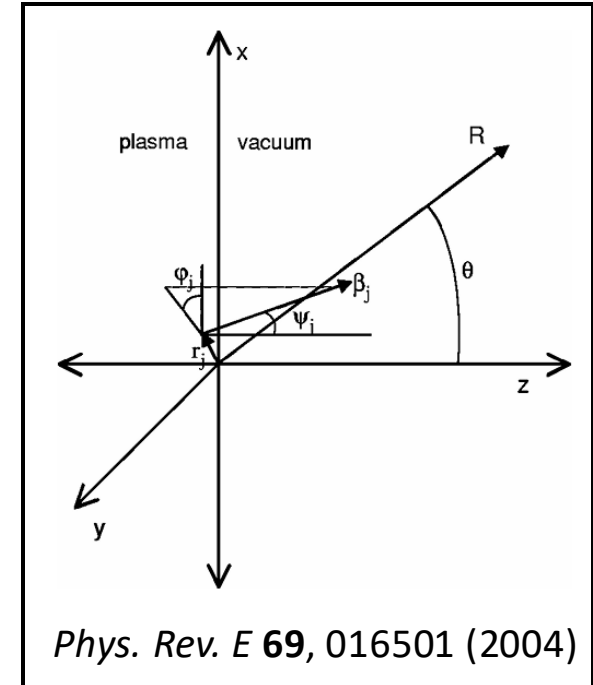


Divergence  $\Leftrightarrow$  Angle of incidence  $\psi \Rightarrow$  Far-field Interferometry

$$\text{Field point spread function}^1: E = \frac{e}{\pi\sqrt{c}} \frac{\psi - \theta}{\gamma^{-2} + |\psi - \theta|^2}$$

$$\text{Total E field: } E_{\text{tot}} = E * \underline{h(\mathbf{r}, \mathbf{p})} e^{ikr}$$

6D phase space distribution



Fringes contain info of ...

# Revealing divergence by COTRI

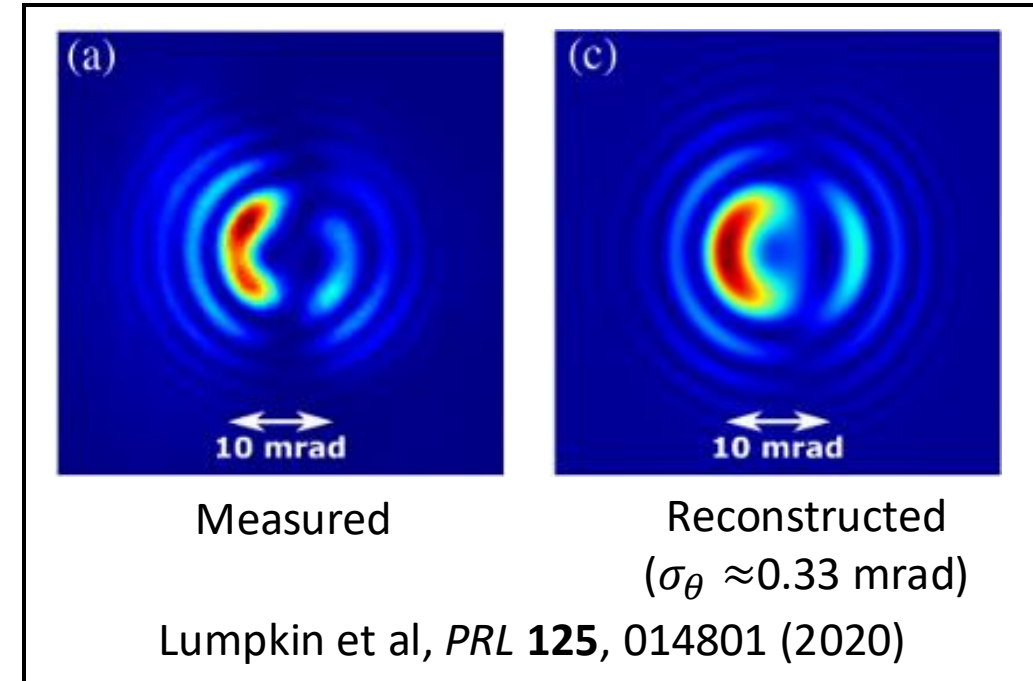
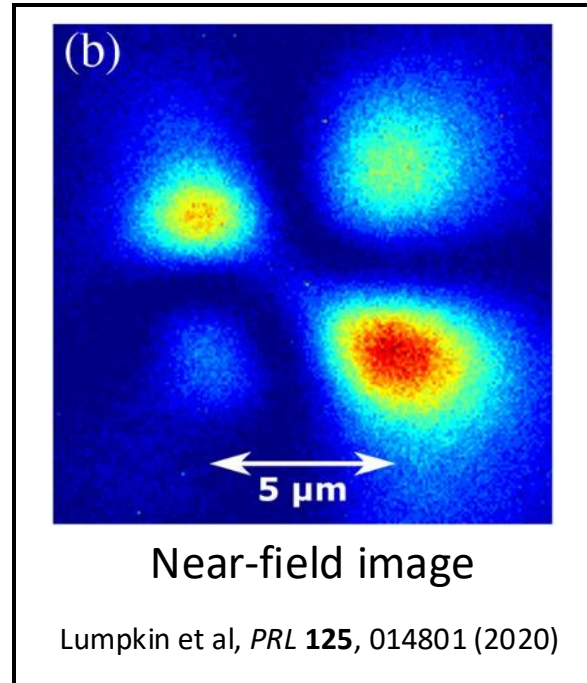
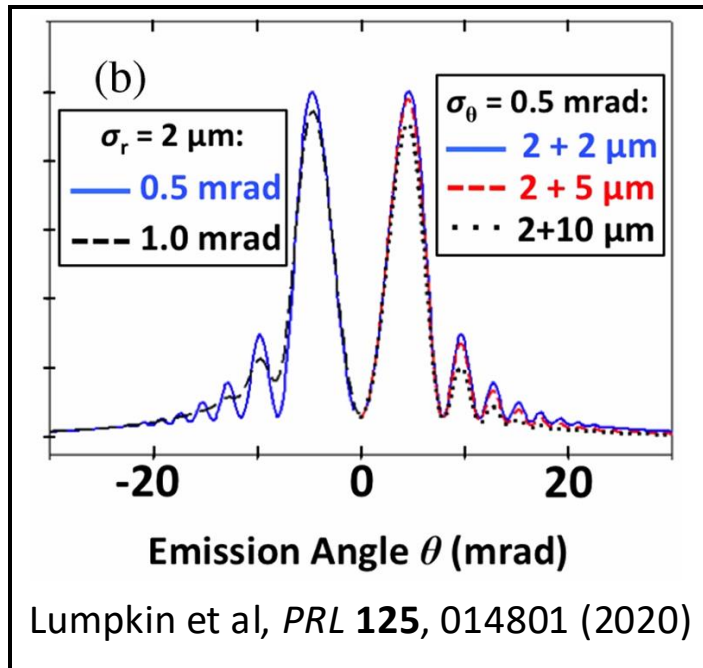
Fringes are sensitive to:

- Optical detection bandwidth  $\Delta\lambda$
- Energy bandwidth  $\Delta\gamma$
- Transverse size  $\sigma_r$
- Divergence  $\sigma_\theta$

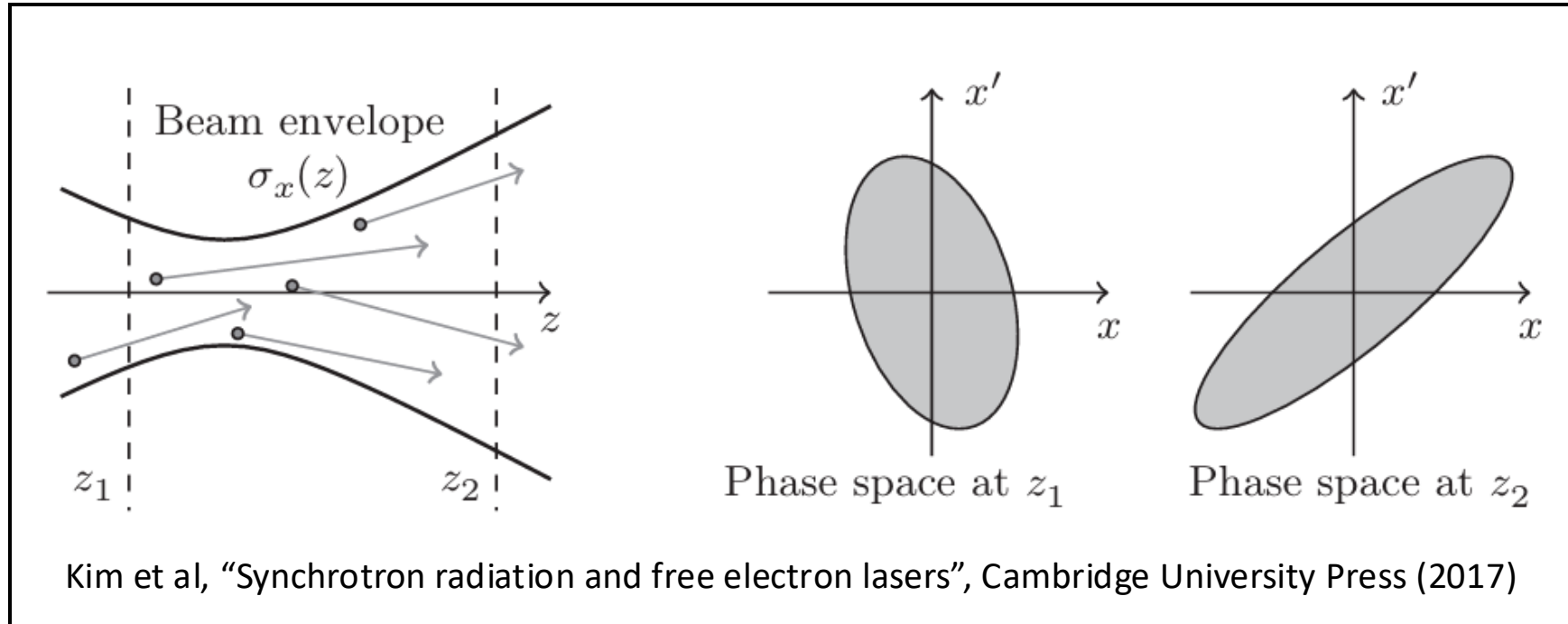
By choosing  $\Delta\lambda$ ,  $\Delta\gamma$ , and  $L$

$\sigma_r$  and  $\sigma_\theta$  can be dominant

Transverse divergence could be revealed!



# Quasi-6D structures explored by COTR(I)



So far, we have obtained the 5D structures:

- 3D density profile (by COTR)
- 2D transverse divergence (by COTR(I))

With reasonable physical assumptions,  
some phase spaces can be ruled out<sup>1</sup>

eg: microbunched portion have  
lower divergence

Obtain an **upper limit** on transverse  
emittance on each slice (quasi-1D)

# Outline

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Introduction to LWFA and its diagnostics

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COTR(I) and quasi-6D structure of e- bunches

3

Future directions, experimental work & conclusion

- Measurement of form factor
- Extension to Smith-Purcell Radiation
- Monitoring the microbunched e- in Free Electron Lasers
- Combination with Diffraction Radiation

# Measurement of form factor

$$\frac{d^2 W_N}{d\omega d\Omega} = [N + N(N - 1) \cdot |F(\omega, \theta)|^2] \cdot \frac{d^2 W_1}{d\omega d\Omega}$$

$$|F(\omega, \theta)| \approx |F_z(\omega, \theta)|$$

$$F(\omega, \theta) = \int \rho(\mathbf{r}) e^{i\mathbf{k}\mathbf{r}} d\mathbf{r} \text{ (Form factor)}$$

With inverse Fourier transform:

With longitudinal and transverse profile separatable:

$$\rho_z(z) = \frac{1}{2\pi} \int F(\omega, \theta) e^{\frac{i\omega z}{c}} d\omega$$

$$F(\omega, \theta) = F_{\perp}(\omega, \theta) F_z(\omega, \theta) = \int \rho_{\perp}(\mathbf{r}_{\perp}) e^{i\mathbf{k}_{\perp}\mathbf{r}_{\perp}} d\mathbf{r}_{\perp} \int \rho_z(z) e^{ik_z z} dz$$

Suppose the e- bunch takes a bi-Gaussian shape:

$$\rho(\mathbf{r}) = \rho_{\perp}(\mathbf{r}_{\perp}) \rho_z(z) = \frac{1}{\sqrt{2\pi}^3 \sigma_{\perp}^2 \sigma_z} e^{-\frac{r_{\perp}^2}{2\sigma_{\perp}}} e^{-\frac{z^2}{2\sigma_z}}$$

$$\text{We have } |F_{\perp}(\omega, \theta)| = e^{-2\pi^2 \frac{\sigma_{\perp}^2}{\lambda^2} \sin^2 \theta} \text{ (close to unity if } \sigma_{\perp} \ll \gamma \lambda)^1$$

$$|F_z(\omega, \theta)| = e^{-2\pi^2 \frac{\sigma_z^2}{\lambda^2} \cos^2 \theta}$$

- **With the knowledge of form factor, we can reconstruct the longitudinal profile of the e- beam.**
- The only general method to go down to sub-fs resolution

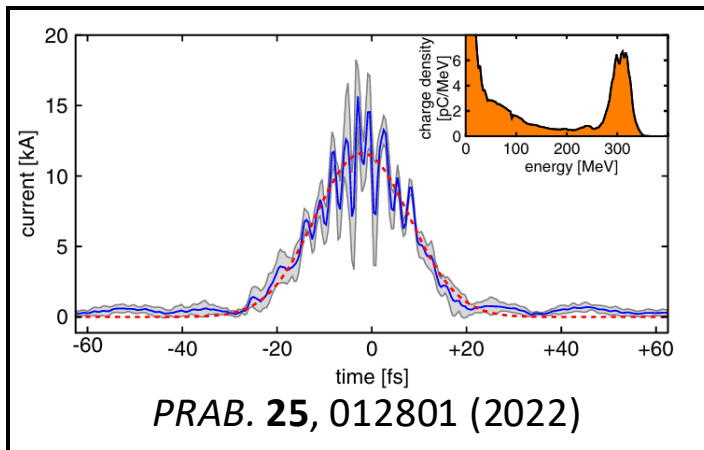
# Measurement of form factor: complex value

$F(\omega, \theta)$  is a complex value:  $\rho_z(z) = \frac{1}{2\pi} \int F(\omega, \theta) e^{\frac{i\omega z}{c}} d\omega$

Measurement of the absolute value<sup>1,2</sup>

$$|F(\omega, \theta)| = \frac{\frac{dW_N}{d\omega} \cdot \frac{dW_1}{d\omega} - N \frac{dW_1}{d\omega}}{N(N-1)}$$

1. Interpolation & extrapolation
2. Phase retrieval algorithm
3. Physical constraints



Measurement of the phase:

**The phase is closely related to the phase of E field**

$$\mathbf{E}_{\text{tot}}(\omega, \theta) = \int \text{FPSF}(\omega, \theta) \rho_z(z) e^{i\frac{\omega z}{c} \cos\theta} dz$$

$|\mathbf{E}_{\text{tot}}(\omega)|$  is captured by the camera, phase  $\varphi(\omega)$ ?



To build **phase-sensitive** detectors!

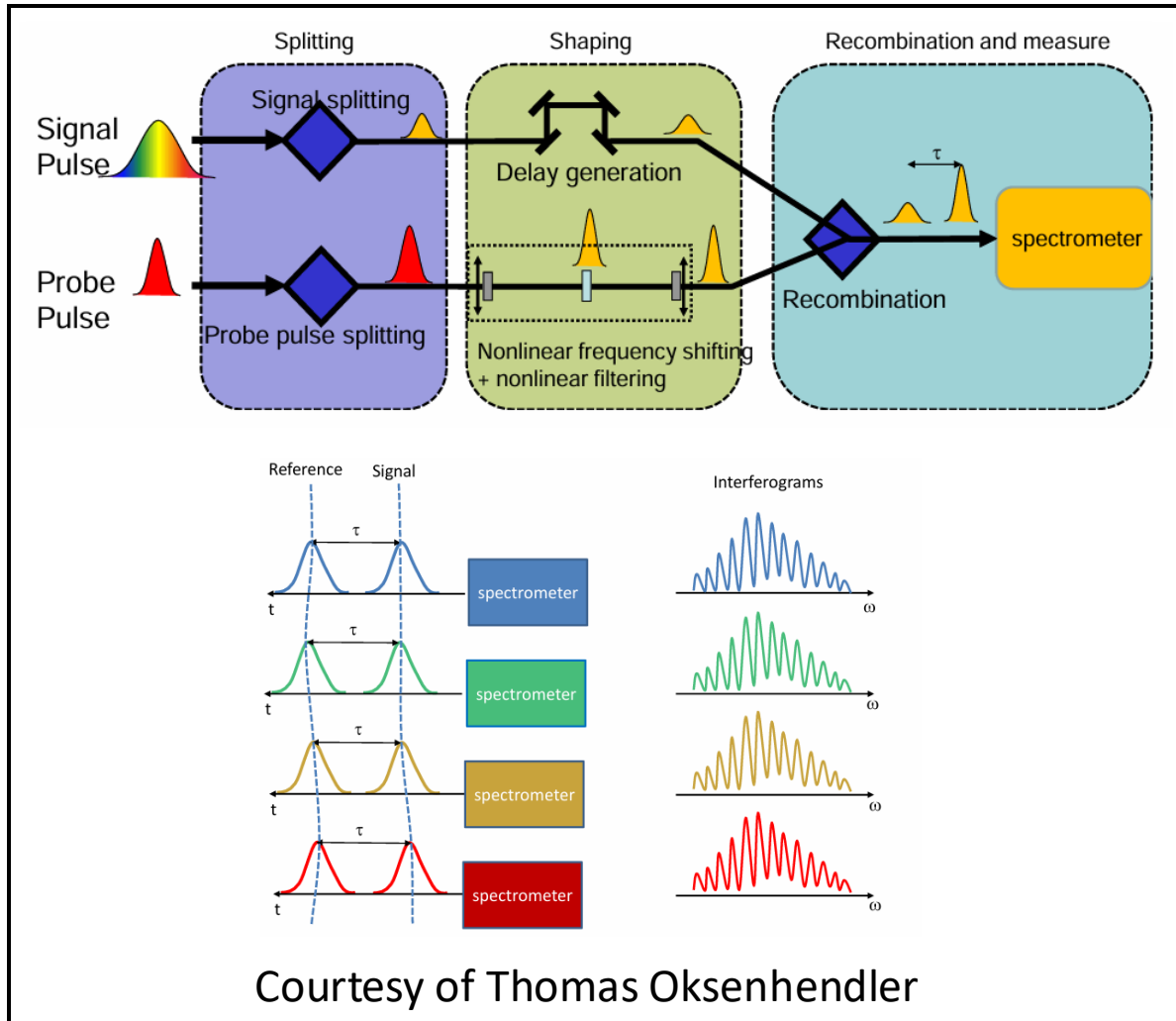
1 Lai et al, *Phys. Rev. E* **50**, 5 (1994)

2 Lai et al, *Phys. Rev. E* **50**, 6 (1994)



# Measurement of form factor: spectral interferometry

## Self-referenced spectral interferometry<sup>1</sup>



$\tilde{E}_{\text{ref}}$  is well characterized in amplitude and phase<sup>2</sup>

**How to detect  $\tilde{E}_{\text{sig}}$ ? From Interferometry**

$$\tilde{S}(\omega) = |\tilde{E}_{\text{ref}} + \tilde{E}_{\text{sig}}|^2 = \tilde{S}_0(\omega) + \tilde{f}(\omega)e^{i\omega\tau} + \tilde{f}^*(\omega)e^{-i\omega\tau}$$

$$\tilde{S}_0(\omega) = |\tilde{E}_{\text{ref}}|^2 + |\tilde{E}_{\text{sig}}|^2 \text{ (DC term)}$$

$$\tilde{f}(\omega) = \tilde{E}_{\text{ref}}\tilde{E}_{\text{sig}}^* \text{ (AC term)}$$

$$|\tilde{E}_{\text{ref}}(\omega)| = \frac{1}{2} \left( \sqrt{\tilde{S}_0(\omega) + 2|\tilde{f}(\omega)|} + \sqrt{\tilde{S}_0(\omega) - 2|\tilde{f}(\omega)|} \right)$$

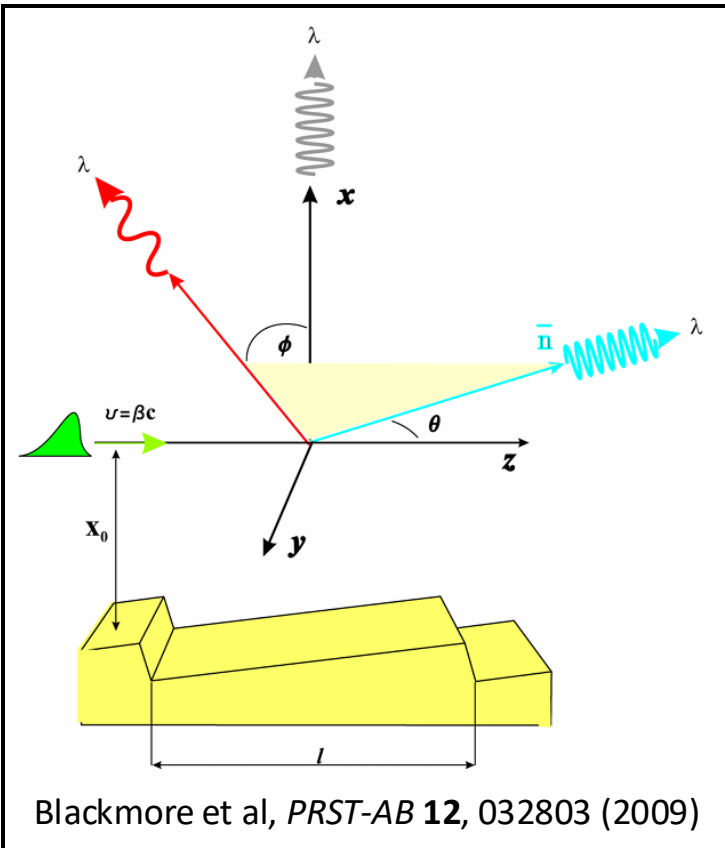
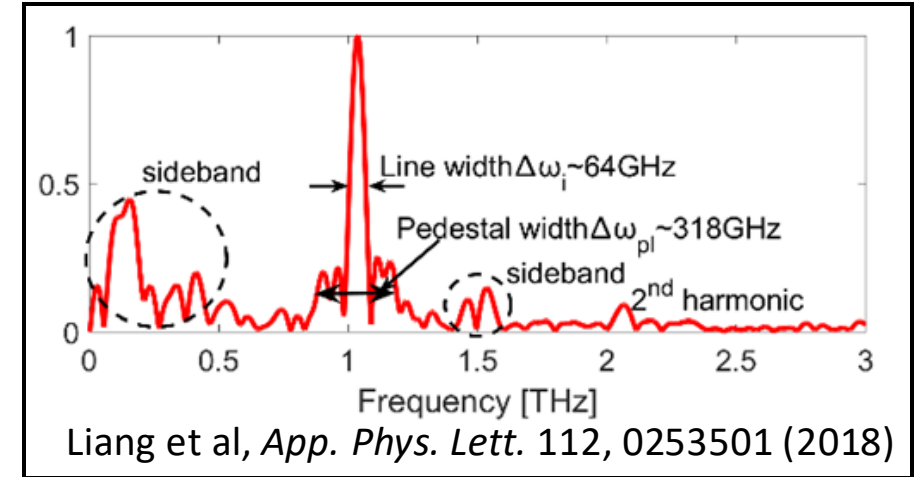
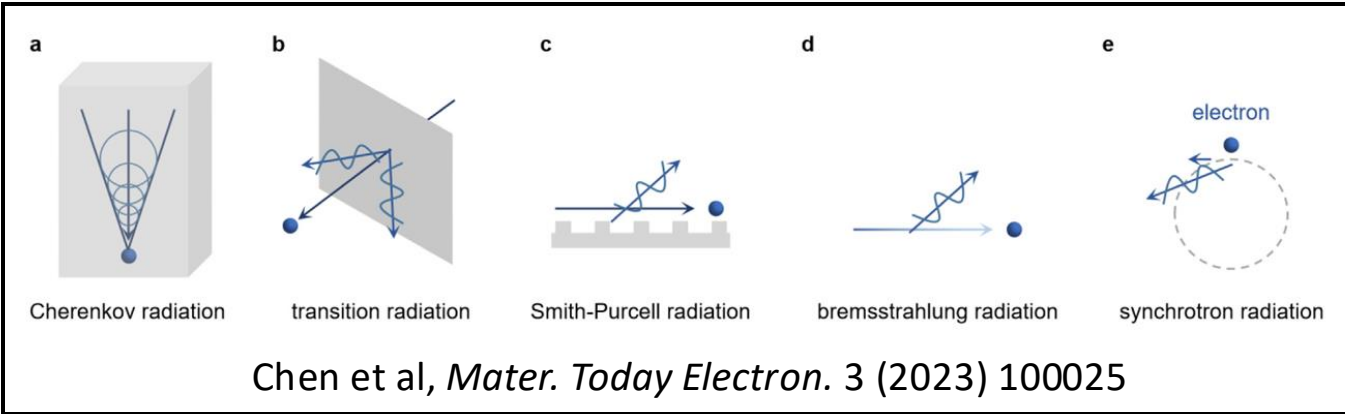
$$|\tilde{E}_{\text{sig}}(\omega)| = \frac{1}{2} \left( \sqrt{\tilde{S}_0(\omega) + 2|\tilde{f}(\omega)|} - \sqrt{\tilde{S}_0(\omega) - 2|\tilde{f}(\omega)|} \right)$$

$$\varphi_{\text{sig}}(\omega) = \varphi_{\text{ref}}(\omega) - \arg(\tilde{f}(\omega))$$

<sup>1</sup> Oksenhendler et al, *Appl. Phys. B* **99**, 7-12 (2001)

<sup>2</sup> Pariente et al, *Nat. Photon.* **10**, 547-553 (2016)

# Extension to Smith-Purcell radiation



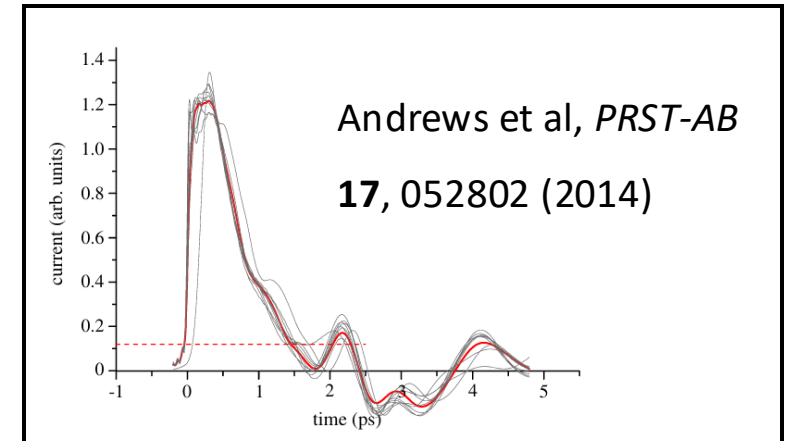
SPR angle-wavelength condition

$$\lambda = \frac{l}{n} \left( \frac{1}{\beta} - \cos\theta \right)$$

$$\frac{dW_1}{d\Omega} = 2\pi e^2 \frac{Z}{l} \frac{n^2 \beta^3}{(1 - \beta \cos\theta)^3} e^{-\frac{2x_0}{\lambda_e} R^2}$$

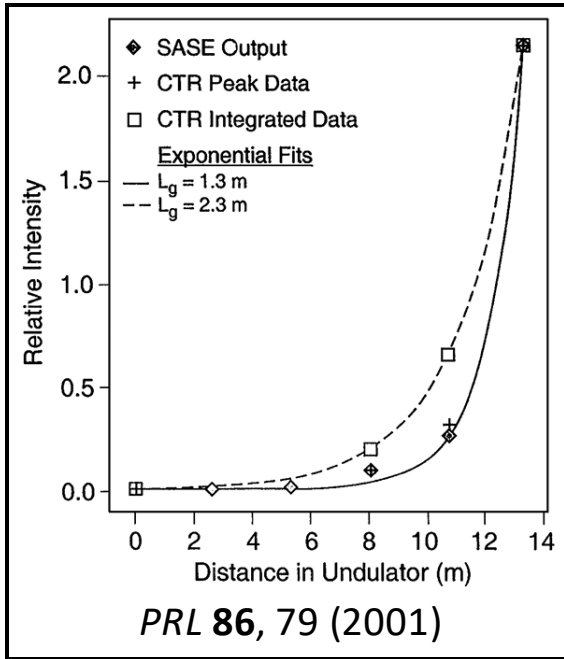
Coherent emission  $\frac{dW_N}{d\Omega} \cong \frac{dW_1}{d\Omega} N^2 S_{\text{coh}}$

where  $S_{\text{coh}} = \left| \int T e^{-i\omega t} dt \right|^2$



1. Another source of THz radiation
2. Possesses microbunching info
3. Help to reveal the temporal profile  
(cross-calibration with COTR) 26

# Monitoring the microbunching in Free Electron Lasers



Seed laser or noise radiation interacting with electrons<sup>1</sup>

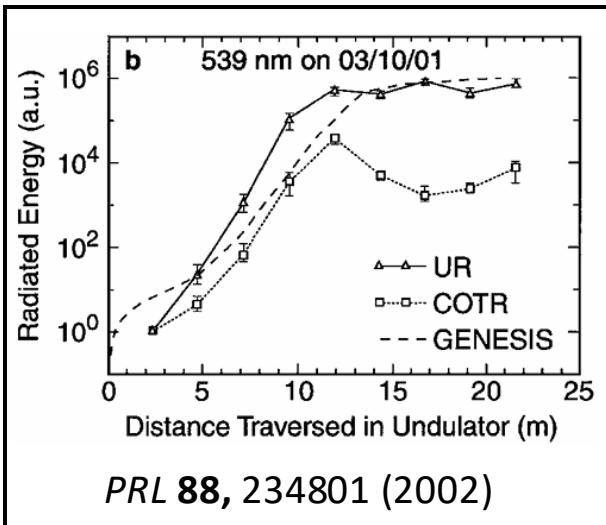
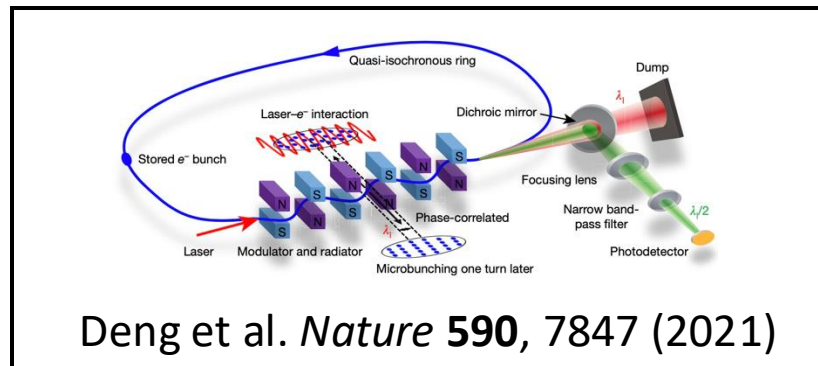
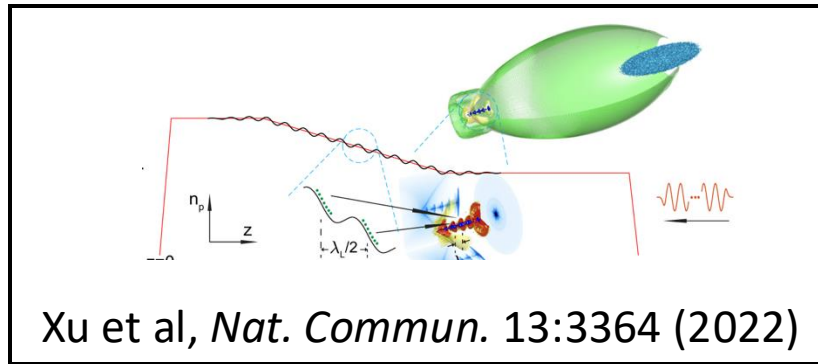
↓

Radiation amplified linearly & e- microbunching growth

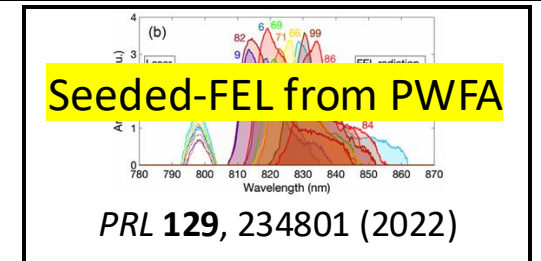
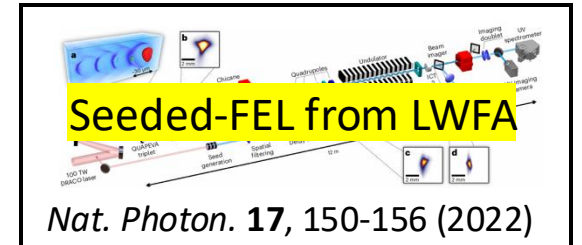
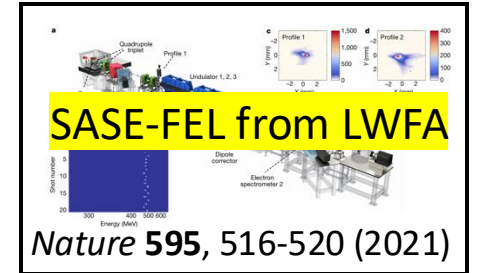
↓

Exponential gain regime & microbunched e-

## Monitoring the pre-microbunching



## Monitoring the microbunching in FEL

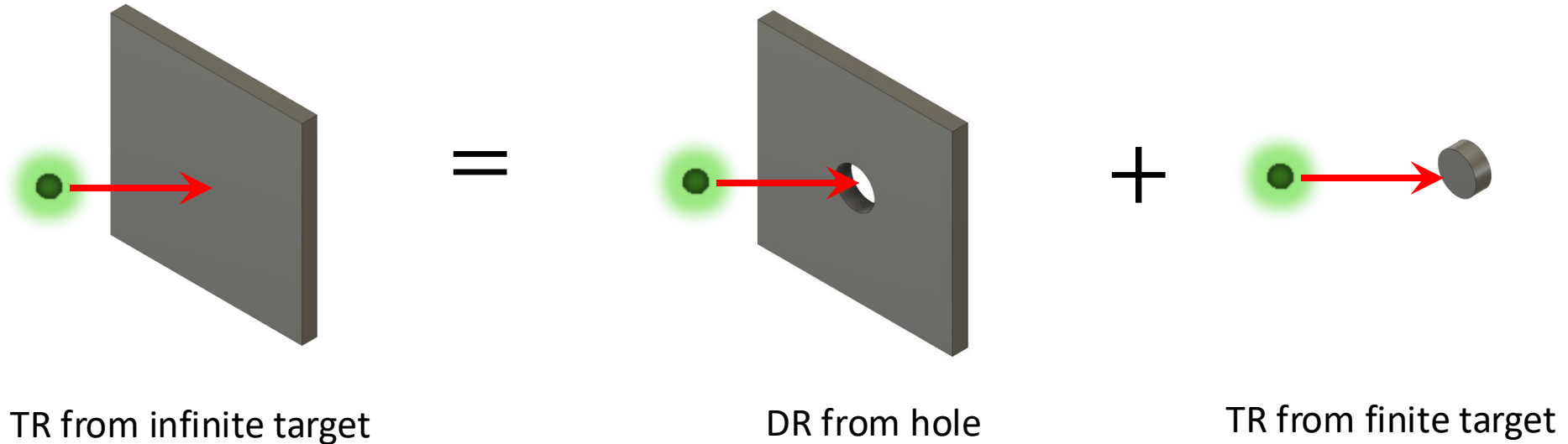


Invasive?

# Combined with diffraction radiation (DR)<sup>1</sup>

Single-shot & **Non-invasive** diagnostics

Babinet's principle<sup>2</sup>:



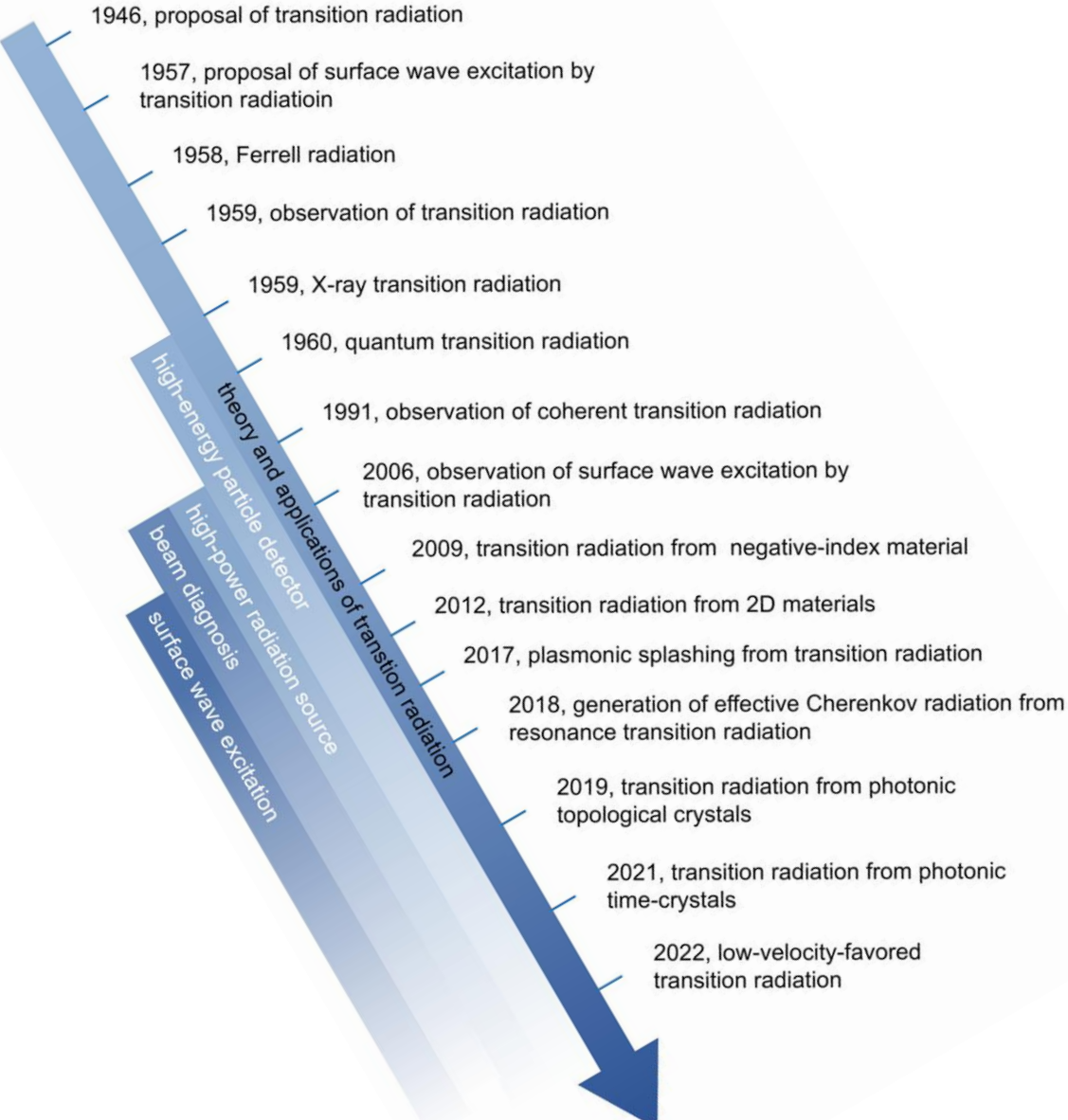
TR from a finite screen can be analytically calculated

$$E_{x,y}^{li}(x_s, y_s, \omega) = -\frac{ie^{ika}}{\lambda a} e^{ik\frac{x_l^2+y_l^2}{2a}} \int dx_s dy_s E_{x,y}^s e^{-ik\frac{x_l x_s + y_l y_s}{2a}} e^{ik\frac{x_s^2+y_s^2}{2a}}$$

<sup>1</sup> Potylitsyn et al. *Diffraction Radiation from Relativistic Particles*, Springer (2010)

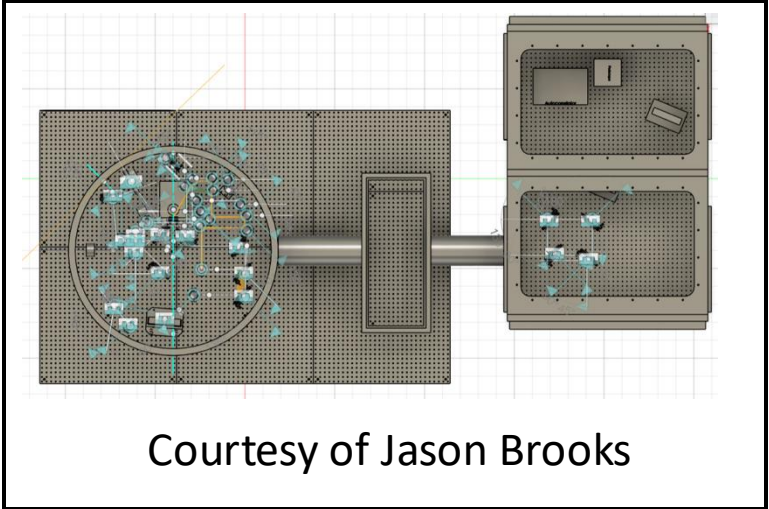
<sup>2</sup> Fiorito et al, *Proceedings of BIW08*, 316-322 (2008)

# Upcoming Experimental Work & Conclusion



Chen et al, *Mater. Today Electron.* 3 (2023) 100025

Future experimental COTR(I) work is scheduled in UT<sup>3</sup> lab.



## Conclusion

1. Introduction on LWFA, and COTR-related diagnostics ⇒ quasi-6D structure
2. Several possible directions in the future

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- Xiantao Cheng
- Maxwell LaBerge



Courtesy of Google image & Ross