A numerical study: Revealing the 3D structure of microbunched laser-wakefield-accelerated electrons by Coherent Transition Radiation

Ze Ouyang

Jun. 20th, 2024

Content

1 Introduction

2 Review of theory of transition radiation

3 Bunch duration, phase delay effect, phase ambiguity & spectrum in TR images

4 Revealing 3D e- bunch info by CTR

5 Conclusion

(Journal Club)

Reconstructing 3D structure of microbunched electrons from plasma wakefield based on coherent optical transition radiation[1]

Ze Ouyang

Feb 29th, 2024

A preliminary study on Transition Radiation[1] & Talk with Prof. Downer

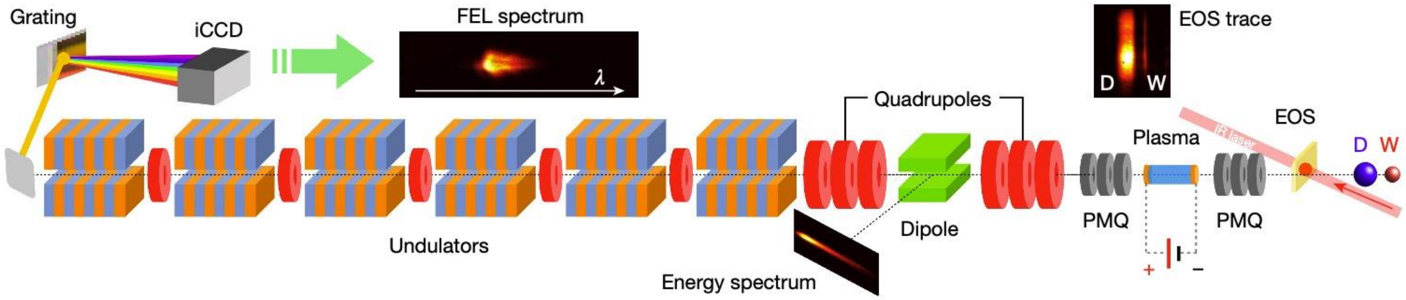
Ze Ouyang

Apr 29th, 2024

Introduction

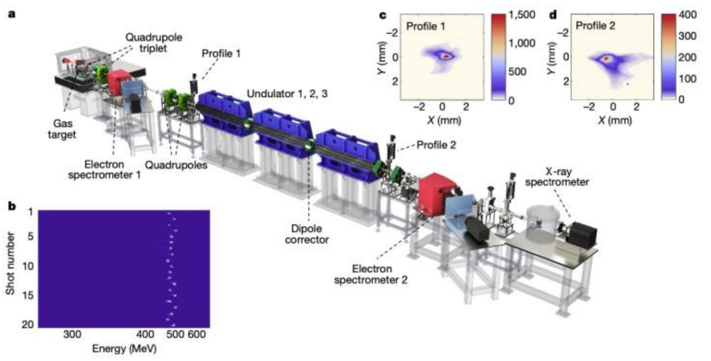
Knowing the 3D structures of microbunched e- beam is crucial for:

1. Understanding the physics of LWFA & PWFA
2. Optimizing the e- beam quality (emittance, energy spread, size)
3. Generating coherent radiation (Synchrotron radiation, secondary radiations & X-FEL)



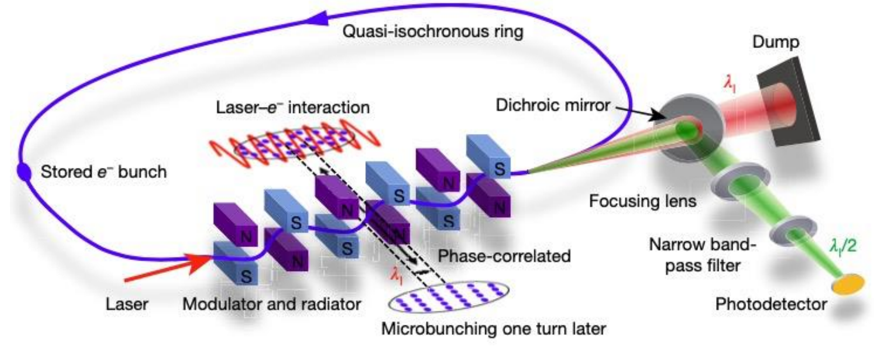
“Tabletop X-FEL based on PWFA”

Pompili et al. *Nature*, **605**, 7911 (2022)



“Tabletop X-FEL based on LWFA”

Wang et al. *Nature*, **595**, 7868 (2021)



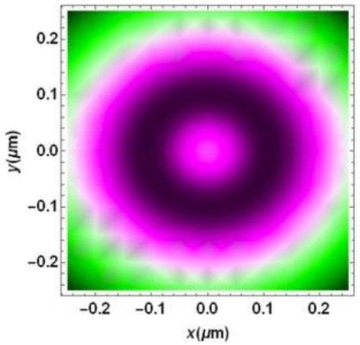
“Steady-state microbunched e- beam storage”

Deng et al. *Nature*, **590**, 7847 (2021)

Introduction[1]

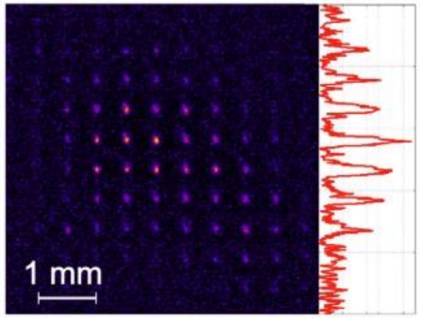
Ways to measure the transverse profile:

1. Radiation-based imaging (TR, SPR, Betatron R)
2. Scintillating screens (phosphor screens)
3. Focus-scans
4. Pepper-pot mask
5. …



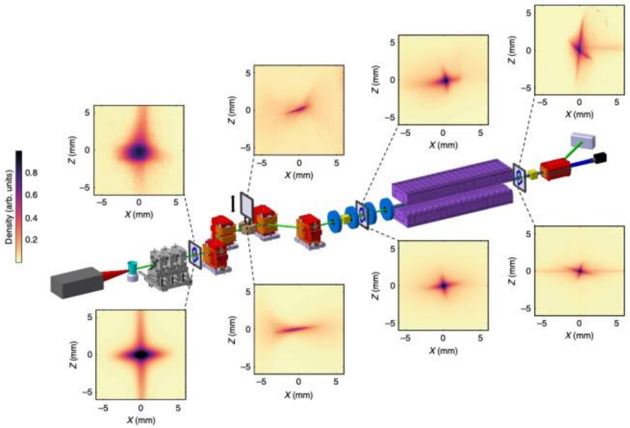
Curcio et al. *Appl. Phys. Lett*, **111**, 133105 (2017)

Brunetti et al. PRL, **105**, 215007 (2010)



Ways to measure the longitudinal profile:

1. Streak cameras
2. Electro-Optic sampling
3. RF deflecting cavities
4. Radiation spectrum
5. …

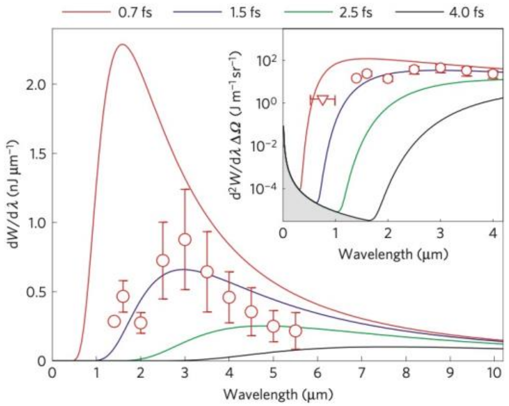


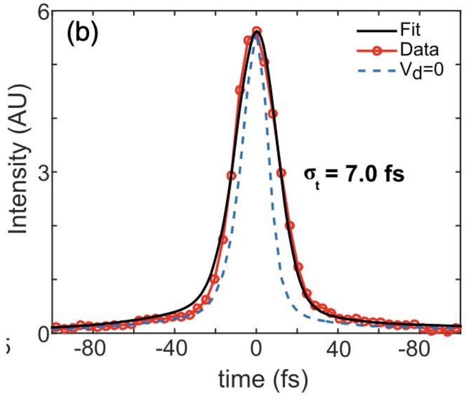
Andre et al. *Nat. Commun*, **9**, 1334 (2018)

1. LWFA ($\lambda_p \geq$10μm)
2. FEL

Microbunched e- beam have much smaller duration.

Lundh et al. *Nat. Phys*, **7**, 3 (2011)

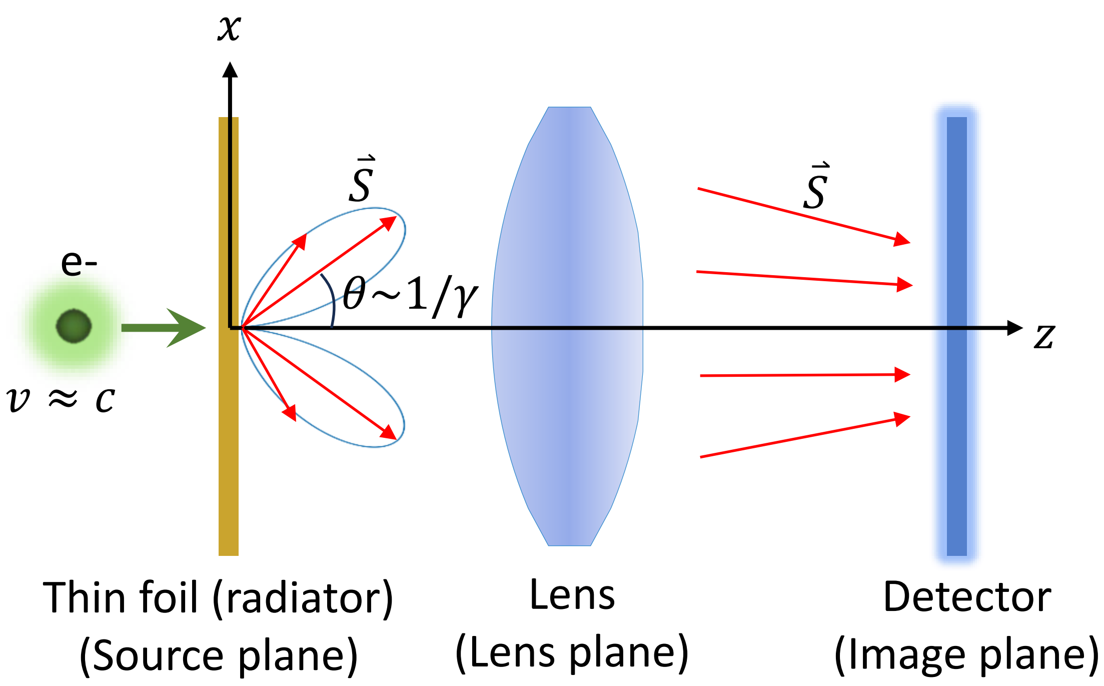




Maxson et al. *PRL*, **118**, 154802 (2017)

1 Downer et al. *RMP*, **90**, 035002 (2018)

Generation of Transition Radiation: single e-



$\boldsymbol{E}$ field on the image plane[1]:

$$\underline{\boldsymbol{E_x}(x_d, y_d)} = \frac{2qk}{Mv} f(\theta_m, \gamma, \zeta) \cos(\varphi) \boldsymbol{e_x}$$

Field PSF, $\mathrm{FPSF}_x(x_d, y_d)$

$$\underline{\boldsymbol{E_y}(x_d, y_d)} = \frac{2qk}{Mv} f(\theta_m, \gamma, \zeta) \sin(\varphi) \boldsymbol{e_y}$$

$\mathrm{FPSF}_x(x_d, y_d)$

where $f(\theta_m, \gamma, \zeta) = \int_0^{\theta_m} \frac{\theta^2}{\theta^2 + \gamma^{-2}} J_1(\zeta\theta) \mathrm{d}\theta$, $\zeta = \frac{kr_d}{M}$, $r_d = \sqrt{x_d^2 + y_d^2}$, M is the magnification, $\tan\varphi = \frac{y_d}{x_d}$, θ_m is the acceptance angle of the lens (or N.A.); $f(\theta_m, \gamma, \zeta) \approx \zeta^{-1}\left(\gamma^{-1}\zeta K_1(\gamma^{-1}\zeta) - J_0(\zeta\theta_m)\right)$ if $\theta_m \gg \frac{1}{\gamma}$.[2]

The Poynting vector is

$$S(x_d, y_d, \omega) = \frac{c}{4\pi^2}\left(|\boldsymbol{E_x}(x_d, y_d)|^2 + \left|\boldsymbol{E_y}(x_d, y_d)\right|^2\right) = \frac{\mathrm{d}^3 I_1}{\mathrm{d}\omega \mathrm{d}x_d \mathrm{d}y_d}$$

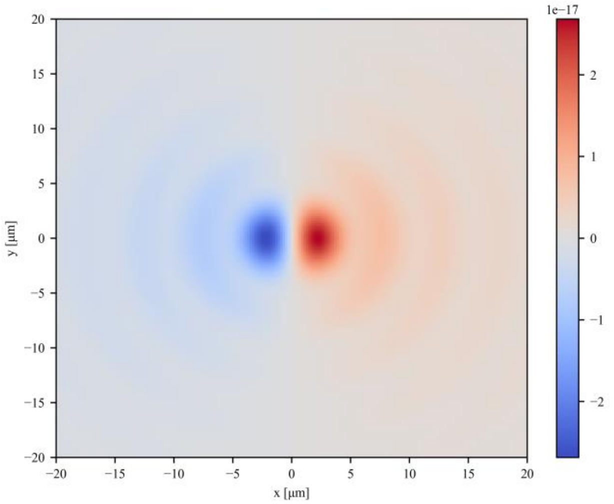
which is also known as Point Spread Function, $\mathrm{PSF}(x_d, y_d)$.

With k(or λ), M, γ, and θ_m given, we can calculate the theoretical distribution of FPSF (x_d, y_d) and $\mathrm{PSF}(x_d, y_d)$ on the image plane.

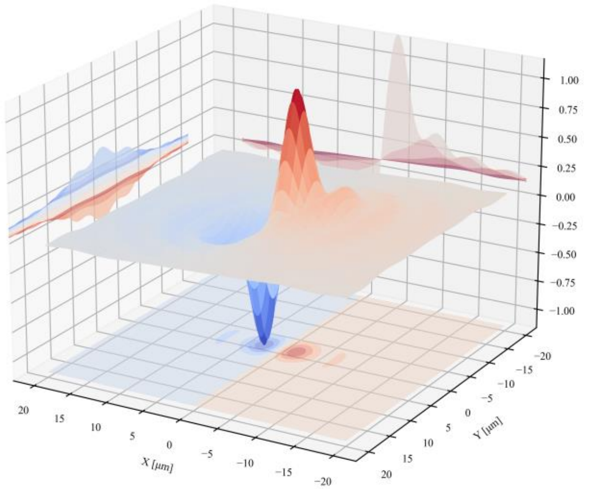
1 Castellano et al. *PRST-AB*, **1**, 062801 (1998)
2 Xiang et al. *Nucl. Instrum. Meth. A* **570**, 3 (2007)

Generation of Transition Radiation: single e-

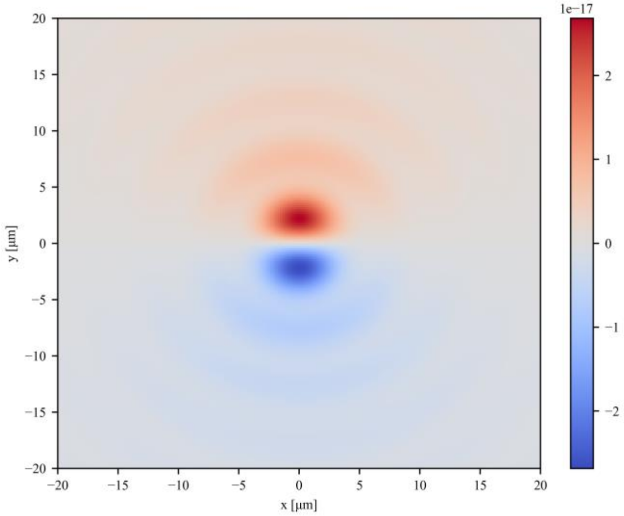
λ=500nm, M=1, γ=391(200MeV), and θ_m=0.1



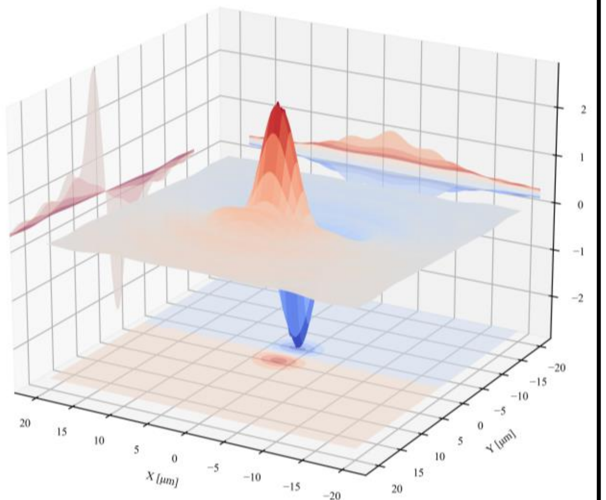
FPSF in the x-distribution



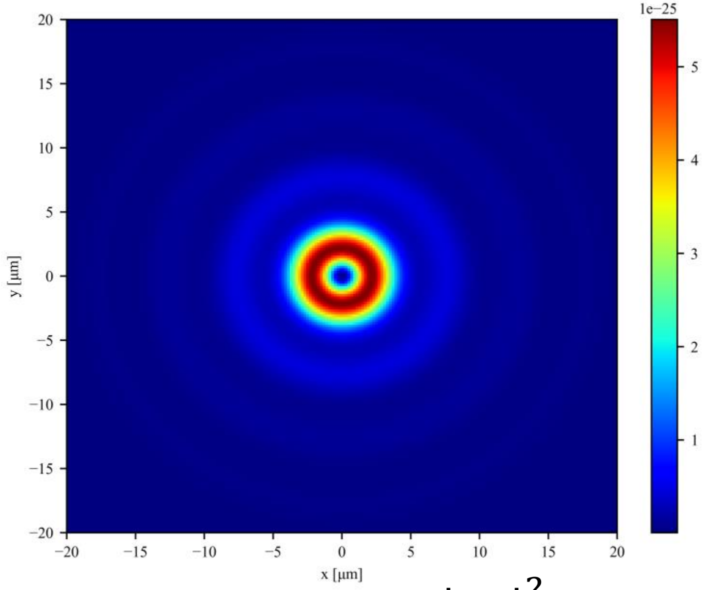
3D version



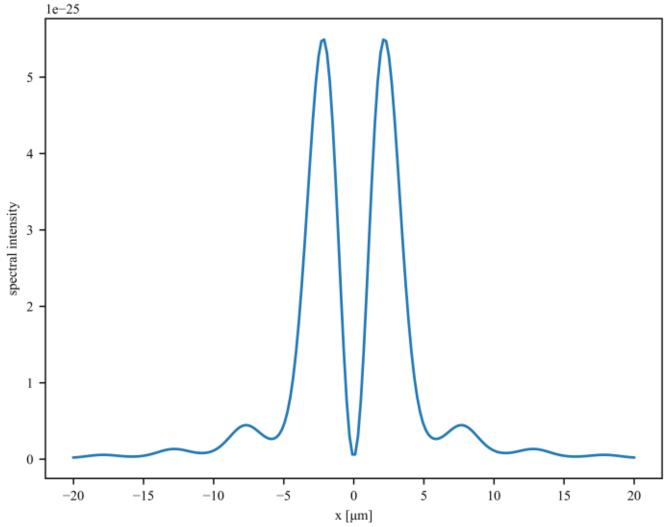
FPSF in the y-distribution



3D version

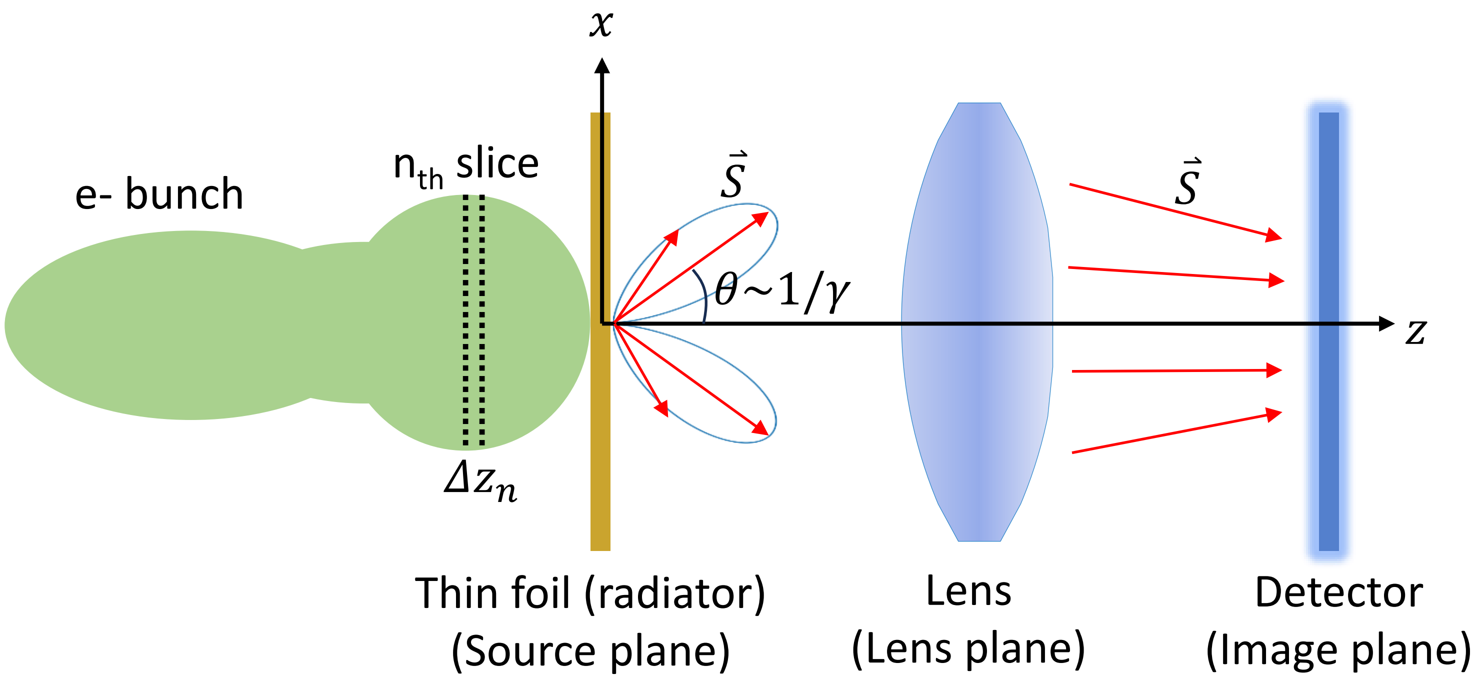


PSF∝($|E_x|^2$+ $|E_y|^2$)



Lineout of PSF at y=0

Generation of Transition Radiation: e- bunch $\rho(x_s, y_s, z_s)$

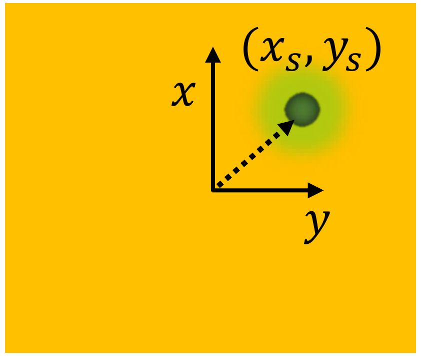


$\Rightarrow$ To obtain $\boldsymbol{E}_{\mathbf{tot}}$

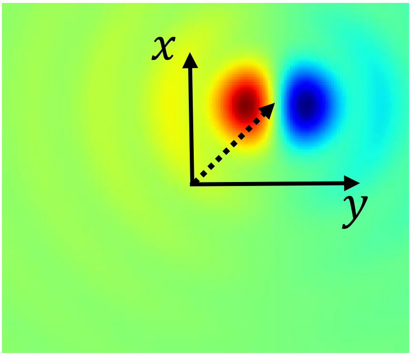
Remark 1:

$\rho(x_s, y_s, z_s)$ gives the number density of electrons in the beam, so $N = \iiint \rho(x_s, y_s, z_s)\, \mathrm{d}x_s \mathrm{d}y_s \mathrm{d}z_s$ gives the total number of electron.

Remark 2:



foil plane



FPSF_x on the image plane will be adjusted to $\mathrm{FPSF}_x(x_d - x_s, y_d - y_s)$

The $\boldsymbol{E}$ field given by the n_{th} slice is

$$\boldsymbol{E}(x_d, y_d) = \boldsymbol{E}_{\boldsymbol{x}}^{(\boldsymbol{n})}(x_d, y_d) + \boldsymbol{E}_{\boldsymbol{x}}^{(\boldsymbol{n})}(x_d, y_d)$$

$$= \underline{\Delta z_n \iint \mathrm{d}x_s \mathrm{d}y_s\, \rho(x_s, y_s, z_n)} \cdot \left(\mathrm{FPSF}_x(x_d - x_s, y_d - y_s) + \mathrm{FPSF}_y(x_d - x_s, y_d - y_s)\right)$$

\# of e- in the slice

Generation of Transition Radiation: e- bunch $\rho(x_s, y_s, z_s)$

- For each slice, there is a phase delay $\exp(-ik\Delta z_n)$, relative to the leading portion of the bunch. Therefore, the total $\boldsymbol{E}$ field is given by

$$\boldsymbol{E}_{\mathbf{tot}}(x_d, y_d) = \iiint \underbrace{\mathrm{d}x_s \mathrm{d}y_s dz_s \cdot \rho(x_s, y_s, z_s)}_{\text{Number of electrons}} \cdot \underbrace{\cos\big(k(z_s - z_u)\big)}_{\text{Phase delay}} \cdot \underbrace{\big(\mathrm{FPSF}_x(x_d - x_s, y_d - y_s) + \mathrm{FPSF}_y(x_d - x_s, y_d - y_s)\big)}_{\text{Field translation}}$$

- It is the $|\boldsymbol{S}|$ rather than $\boldsymbol{E}$ field that the detector records, therefore, the total energy spectral is given by

$$S_{\mathrm{tot}}(x_d, y_d) = \frac{c}{4\pi^2} |\boldsymbol{E}_{\mathbf{tot}}(x_d, y_d)|^2$$

- After simplification, this leads to

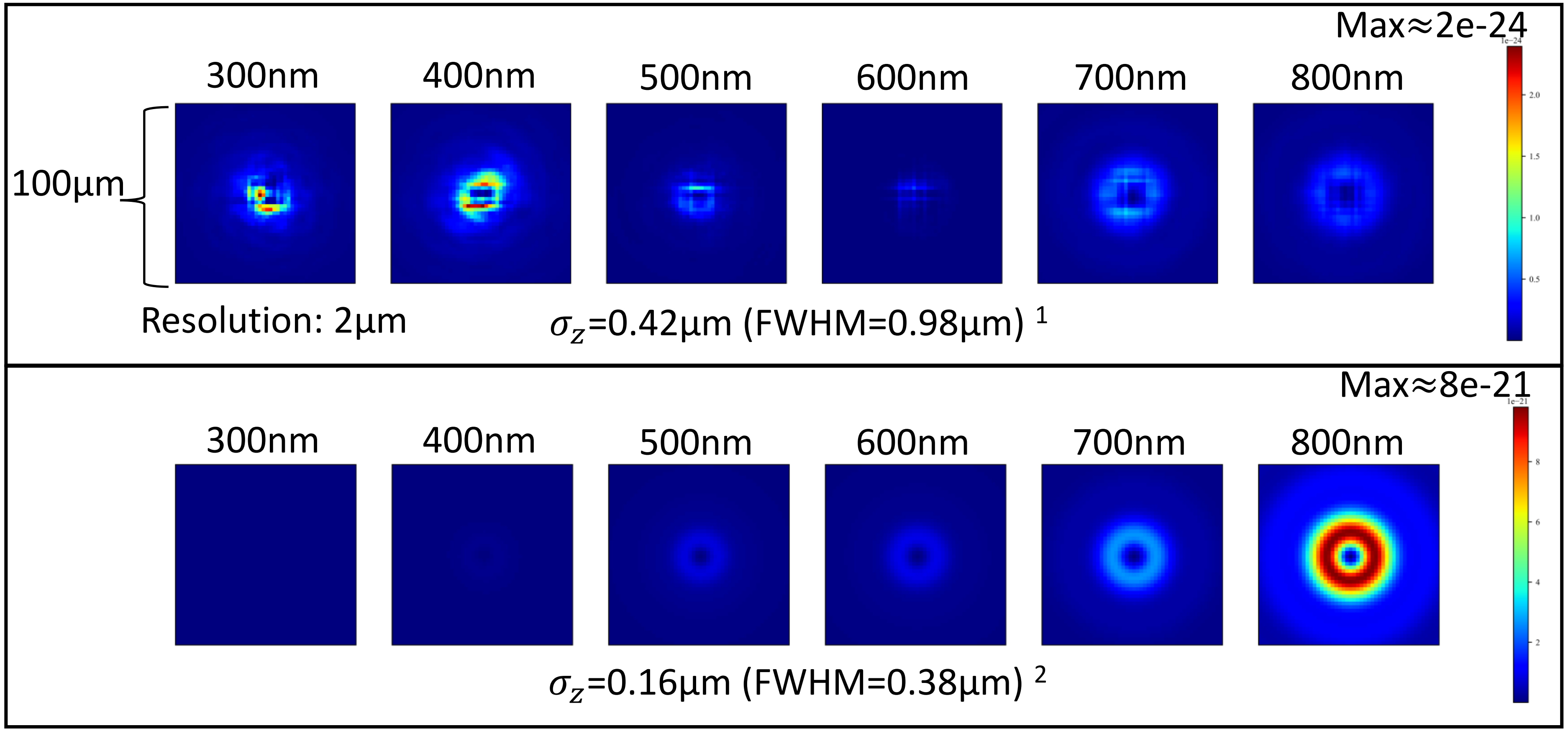
$$S_{\mathrm{tot}}(x_d, y_d) = \frac{c}{4\pi^2}\left(\left|\iiint \mathrm{d}x_s \mathrm{d}y_s dz_s \cdot \rho(x_s, y_s, z_s) \cdot \cos\big(k(z_s - z_u)\big)\, \mathrm{FPSF}_x(x_d - x_s, y_d - y_s)\right|^2 + \left|\iiint \mathrm{d}x_s \mathrm{d}y_s dz_s \cdot \rho(x_s, y_s, z_s) \cdot \cos\big(k(z_s - z_u)\big)\, \mathrm{FPSF}_y(x_d - x_s, y_d - y_s)\right|^2\right)$$

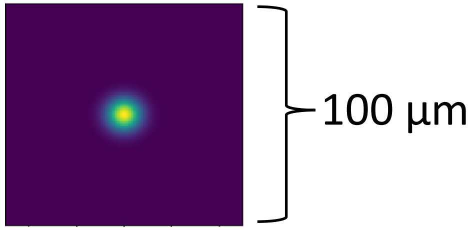
Simulation of Transition Radiation: different e- bunch duration

Set M=10, θ_m=0.28, γ=391(200MeV);

Set e- bunch: $\rho(x_s, y_s, z_s) = N_e \frac{1}{\sqrt{2\pi}\sigma_x} \exp\left(-\frac{(x-\mu_x)^2}{2\sigma_x^2}\right) \frac{1}{\sqrt{2\pi}\sigma_y} \exp\left(-\frac{(y-\mu_y)^2}{2\sigma_y^2}\right) \frac{1}{\sqrt{2\pi}\sigma_z} \exp\left(-\frac{(z-\mu_z)^2}{2\sigma_z^2}\right)$

Params	Value
N_e	1e9
μ_x	0µm
σ_x	5µm
μ_y	0µm
σ_y	5µm
μ_z	0µm
σ_z	0.42 or 0.16µm



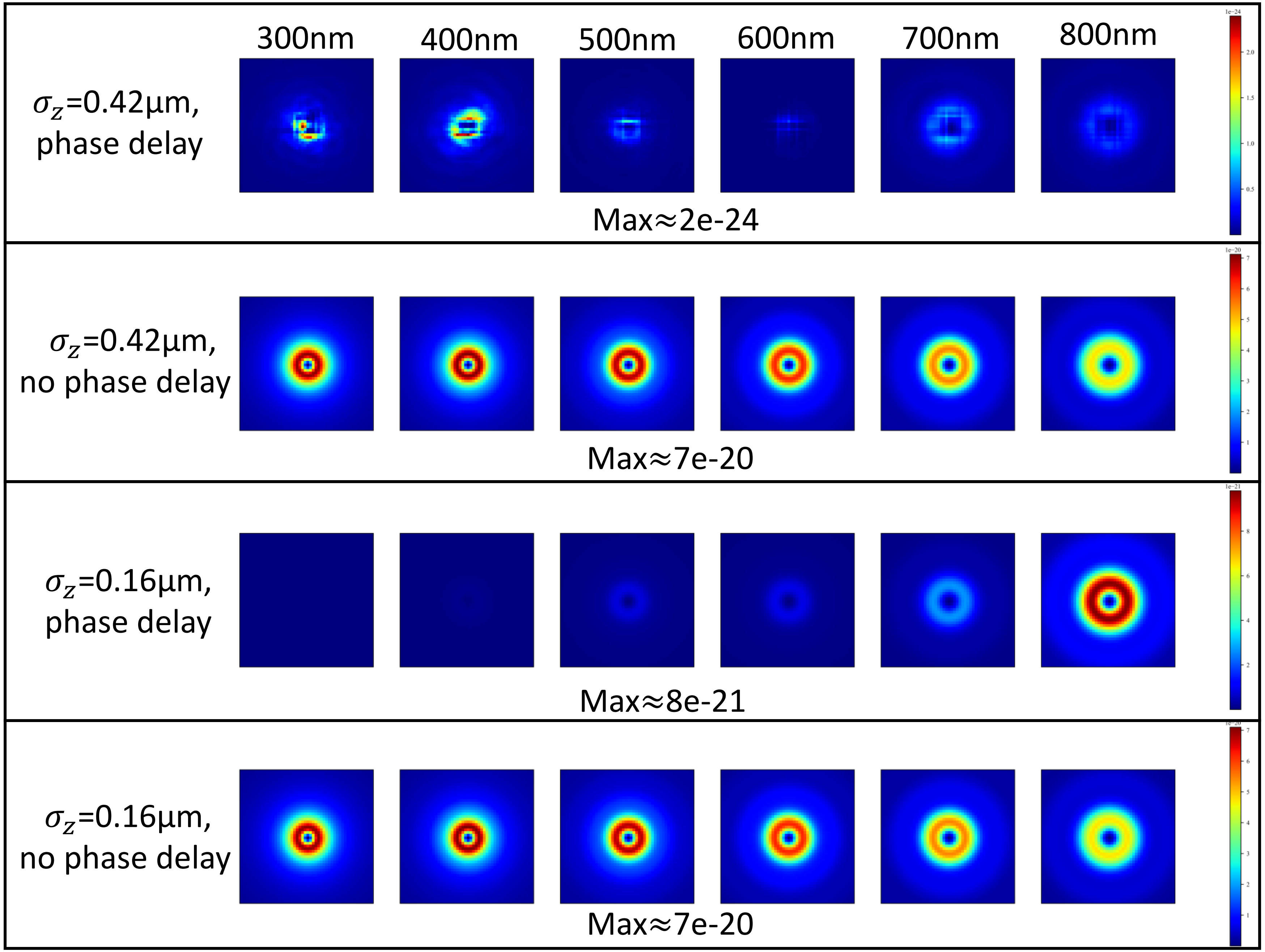


e- bunch in x-y plane

Because of the phase delay effect, only radiation with $\lambda_{rad} > \sigma_z$ is **likely** to be coherent.

1 Lundh et al. *Nat.Phys*, **7**, 3 (2011)
2 LaBerge et al. https://www.researchsquare.com/article/rs-3894996/v1

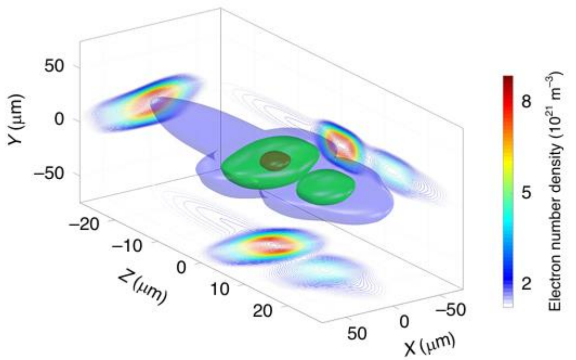
Simulation of Transition Radiation: phase delay



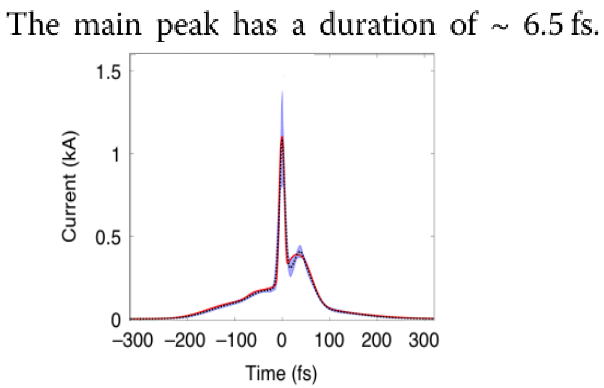
Comments:

1. An important factor to determine the TR intensity when $\lambda_{\text{rad}} < \sigma_z$, or say in incoherent situation
2. Given the fact that e- bunch duration can go down to ~100nm (from LWFA or FEL), it is also important in coherent situation with λ_{rad} in the optical range

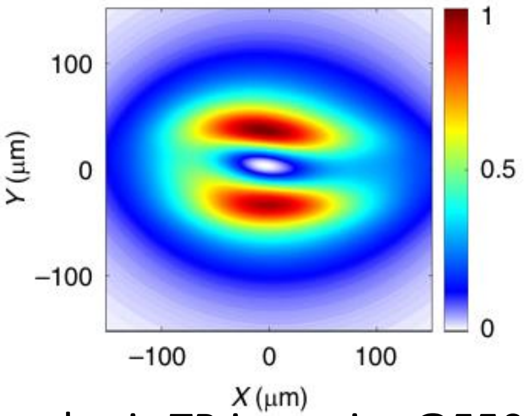
Latest Results in this field[1,2]



3D e- bunch profile

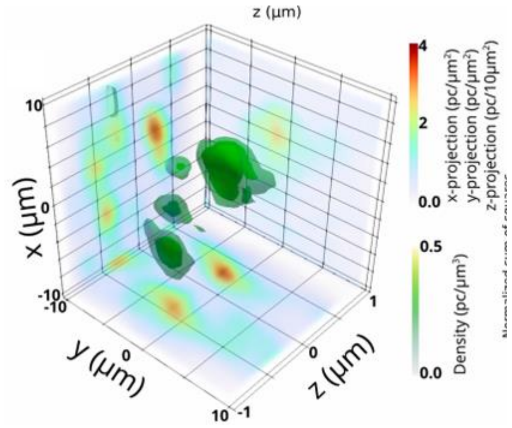


σ_z ≈2.1μm

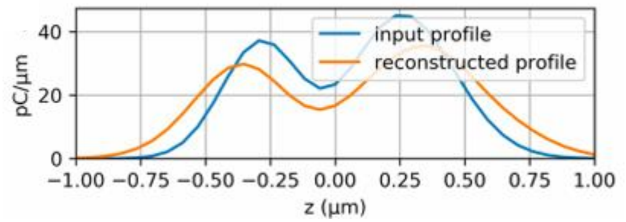


Synthetic TR intensity @550nm

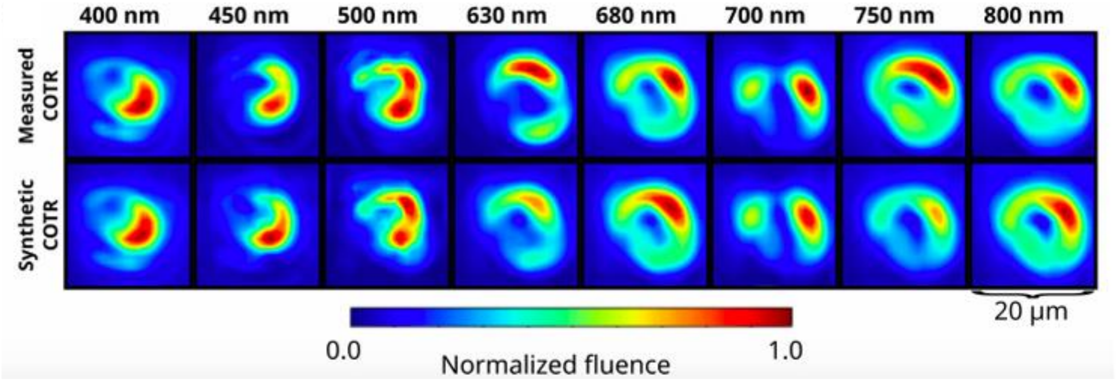
1. No phase delay neither in the main context
2. Nor in the reference (Potylitsyn et al. *PRST-AB*, **23**, 042804 (2020))



3D e- bunch profile



σ_z ≈0.25μm



TR intensity distribution @ different wavelengths

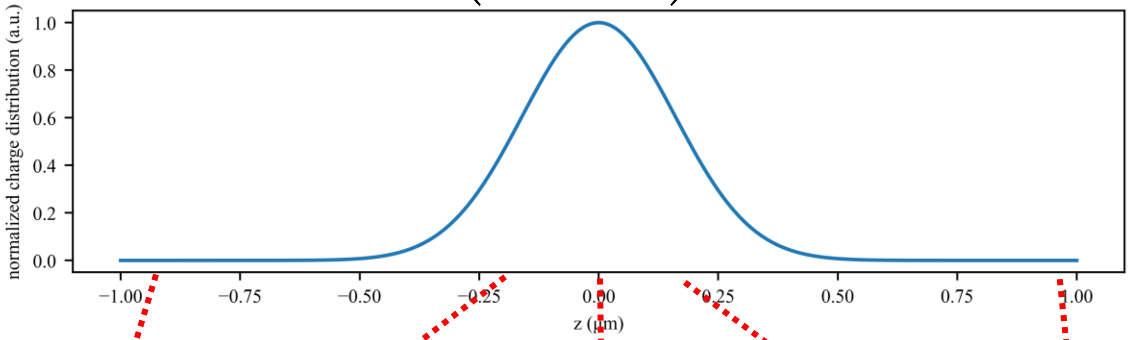
1 Huang et al. *Light sci.appl*, **13**, 1 (2024)
2 LaBerge et al. https://www.researchsquare.com/article/rs-3894996/v1 (2024)

Simulation of Transition Radiation: initial phase position

Set M=10, θ_m=0.28, γ=391(200MeV);

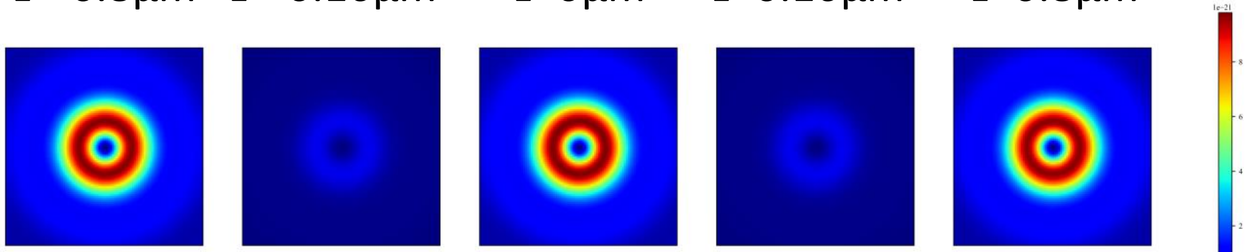
Set e- bunch: $\rho(x_s, y_s, z_s) = N_e \frac{1}{\sqrt{2\pi}\sigma_x} \exp\left(-\frac{(x-\mu_x)^2}{2\sigma_x^2}\right) \frac{1}{\sqrt{2\pi}\sigma_y} \exp\left(-\frac{(y-\mu_y)^2}{2\sigma_y^2}\right) \frac{1}{\sqrt{2\pi}\sigma_z} \exp\left(-\frac{(z-\mu_z)^2}{2\sigma_z^2}\right)$

Params	Value
N_e	1e9
μ_x	0μm
σ_x	5μm
μ_y	0μm
σ_y	5μm
μ_z	0μm
σ_z	0.16μm



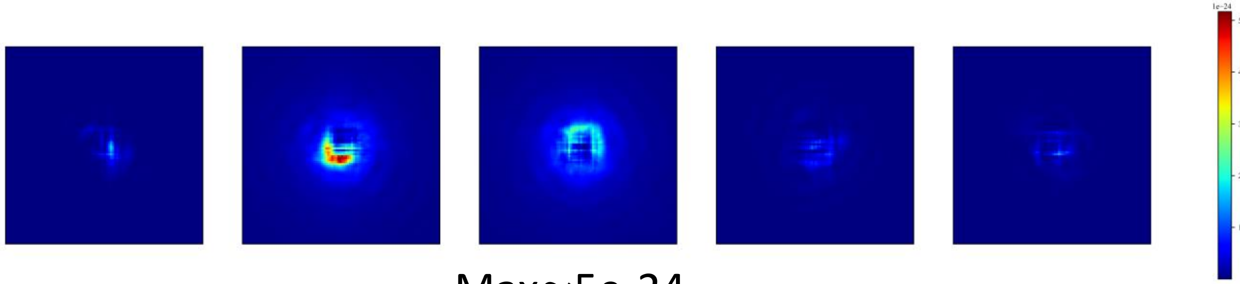
Initial phase at: z=-0.8μm z=-0.16μm z=0μm z=0.16μm z=0.8μm

λ_{rad}=800nm (coherent)

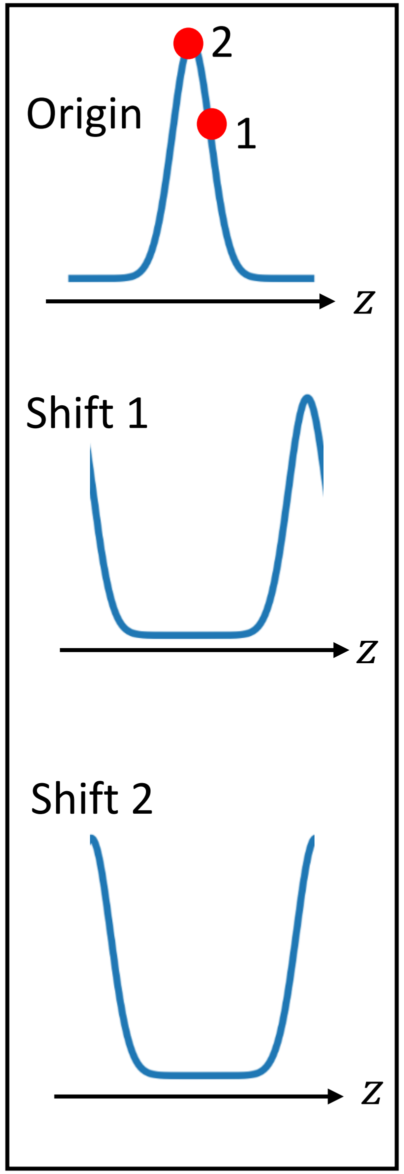


Max≈8e-21

λ_{rad}=300nm (incoherent)



Max≈5e-24



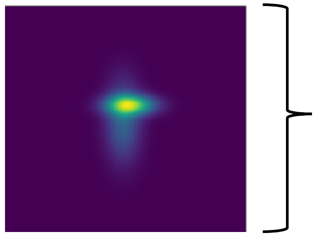
Phase info is inherently ambiguous?

Simulation of Transition Radiation: phase ambiguity

Set M=10, θ_m=0.28, γ=391(200MeV);

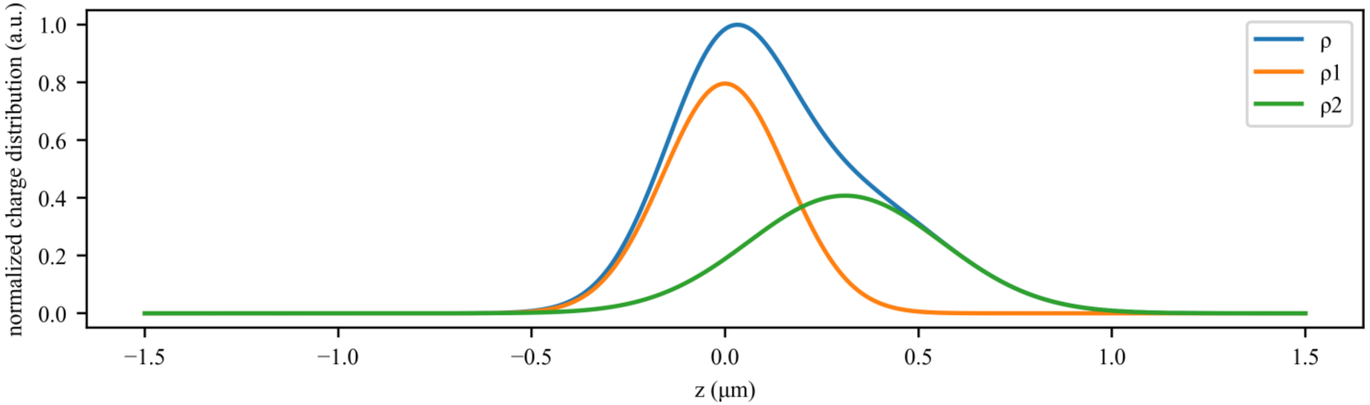
Set e- bunch: $\rho(x_s, y_s, z_s) = \sum_{i=1}^{2} N_{e_i} \frac{1}{\sqrt{2\pi}\sigma_{x_i}} \exp\left(-\frac{\left(x_i-\mu_{x_i}\right)^2}{2\sigma_{x_i}^2}\right) \frac{1}{\sqrt{2\pi}\sigma_{y_i}} \exp\left(-\frac{\left(y_i-\mu_{y_i}\right)^2}{2\sigma_{y_i}^2}\right) \frac{1}{\sqrt{2\pi}\sigma_{z_i}} \exp\left(-\frac{\left(z_i-\mu_{z_i}\right)^2}{2\sigma_{z_i}^2}\right)$

Params	ρ_1	ρ_2
N_e	1e9	0.8e9
μ_x	0µm	3µm
σ_x	5µm	7µm
μ_y	0µm	7µm
σ_y	12µm	3µm
μ_z	0µm	$\pm$0.31µm
σ_z	0.16µm	0.25µm

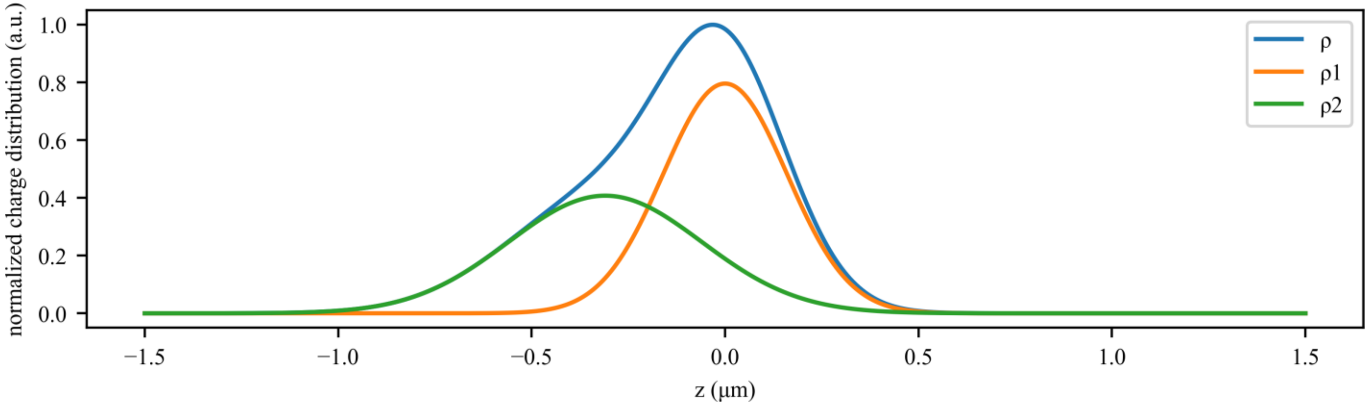


100µm

e- bunch transverse profile

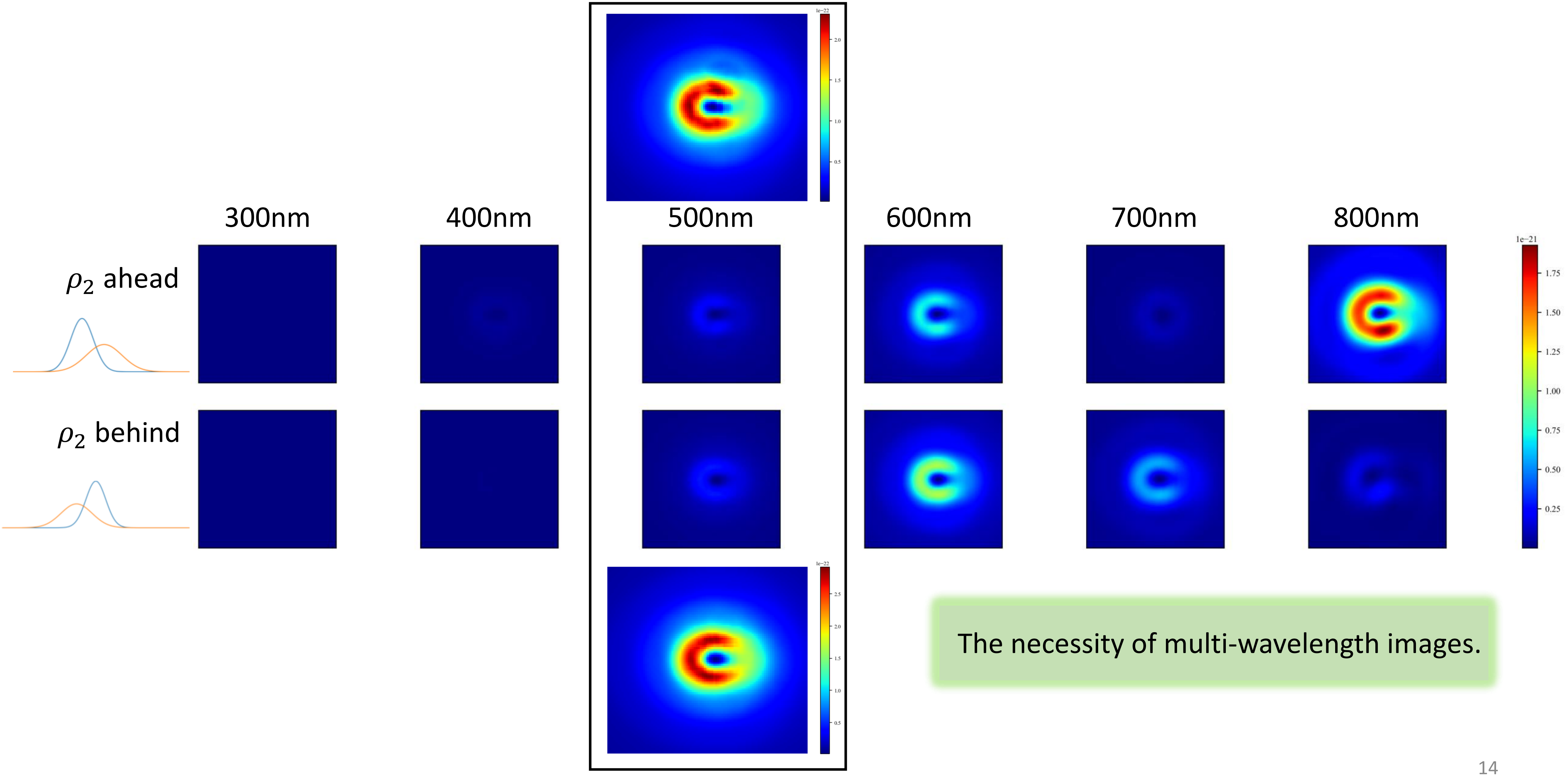


ρ_2 ahead



ρ_2 behind

Simulation of Transition Radiation: phase ambiguity



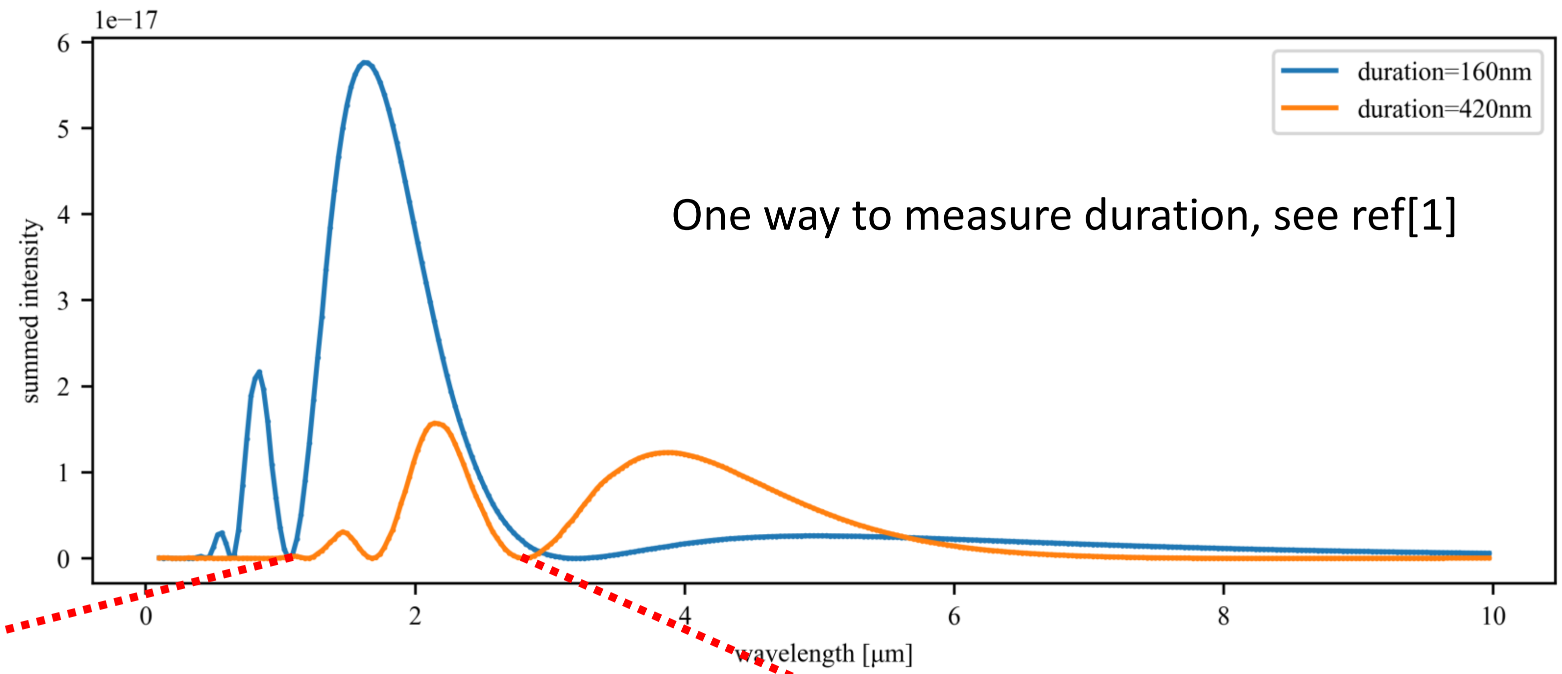
The necessity of multi-wavelength images.

Simulation of Transition Radiation: intensity spectrum

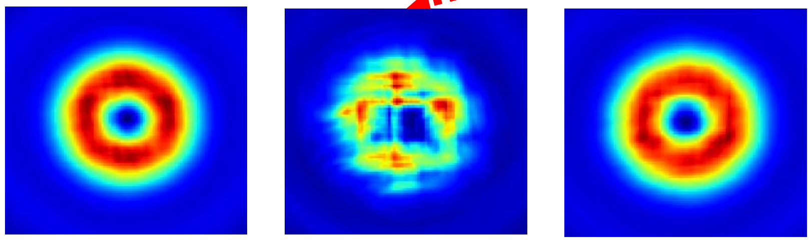
Set M=10, θ_m=0.28, γ=391(200MeV);

Set e- bunch: $\rho(x_s, y_s, z_s) = N_e \frac{1}{\sqrt{2\pi}\sigma_x} \exp\left(-\frac{(x-\mu_x)^2}{2\sigma_x^2}\right) \frac{1}{\sqrt{2\pi}\sigma_y} \exp\left(-\frac{(y-\mu_y)^2}{2\sigma_y^2}\right) \frac{1}{\sqrt{2\pi}\sigma_z} \exp\left(-\frac{(z-\mu_z)^2}{2\sigma_z^2}\right)$

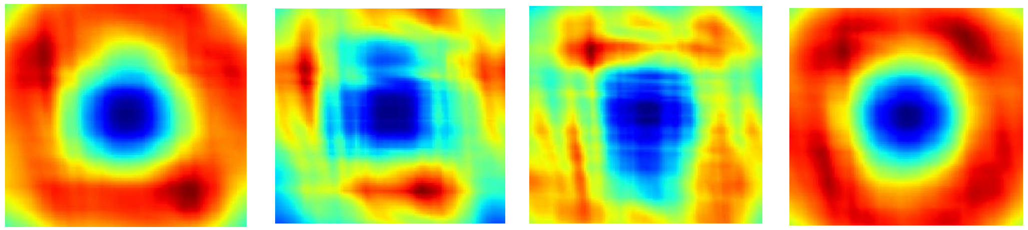
Params	Value
N_e	1e9
μ_x	0μm
σ_x	5μm
μ_y	0μm
σ_y	5μm
μ_z	0μm
σ_z	0.16μm or 0.42μm



One way to measure duration, see ref[1]



λ=1030nm λ=1060nm λ=1090nm

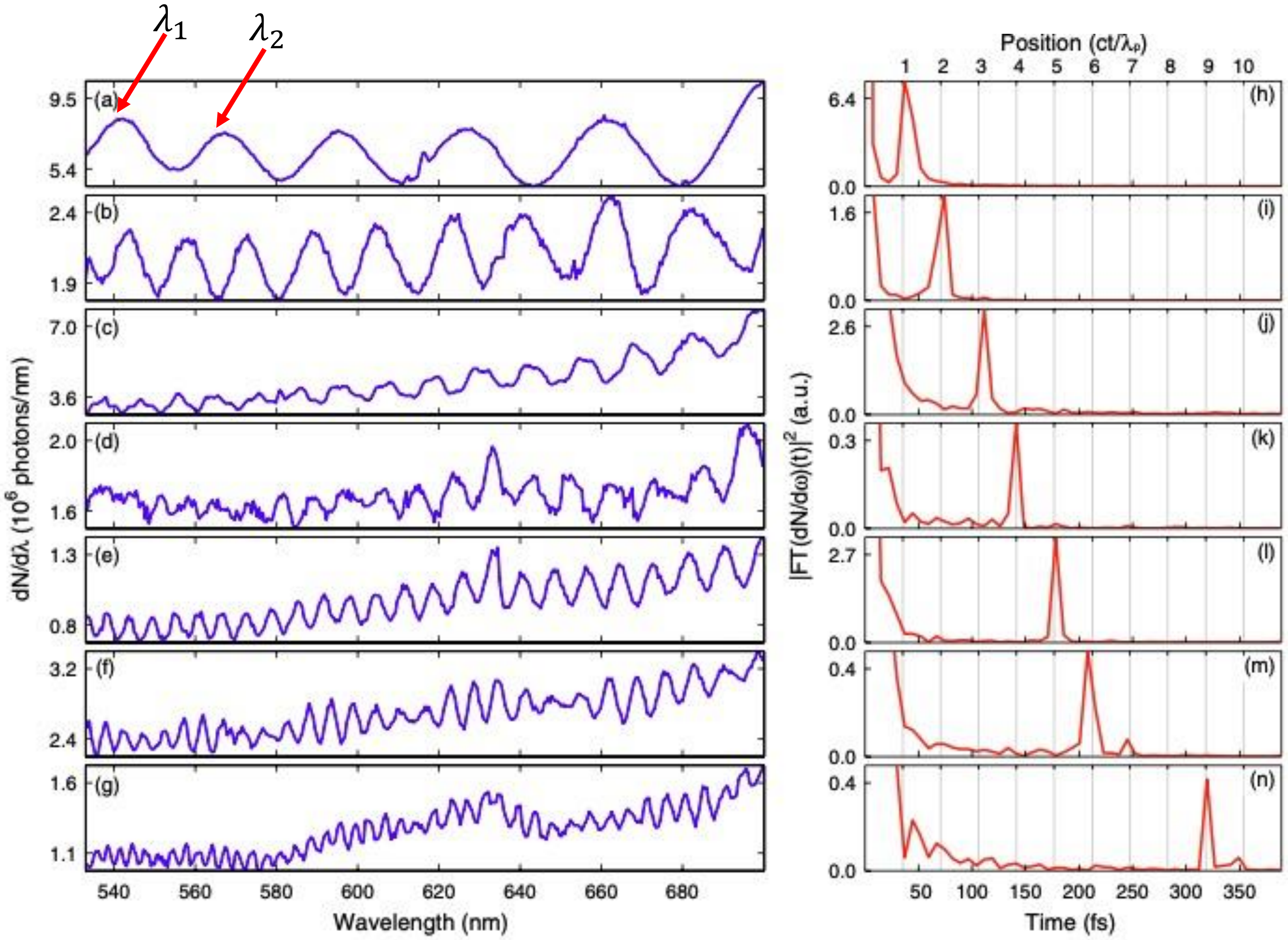


λ=2760nm λ=2792nm λ=2823nm λ=2853nm

Incoherence occurs periodically even at $\lambda_{\rm rad} \gg \sigma$

1 Lundh et al. *Nat.Phys*, **7**, 3 (2011)

Simulation of Transition Radiation: intensity spectrum[1]



$$\frac{L}{\lambda_1} - \frac{L}{\lambda_2} = 1$$

$$\Rightarrow L = \frac{\lambda^2}{\Delta\lambda}$$

"electron bunch train"

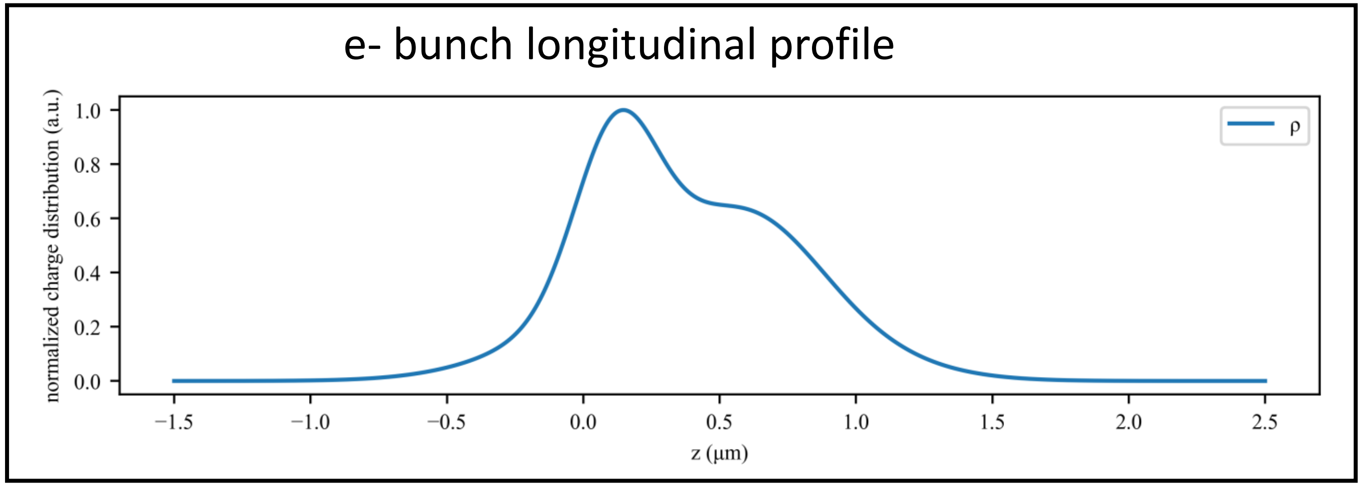
1 Lundh et al. *PRL*, **110**, 065005 (2013)

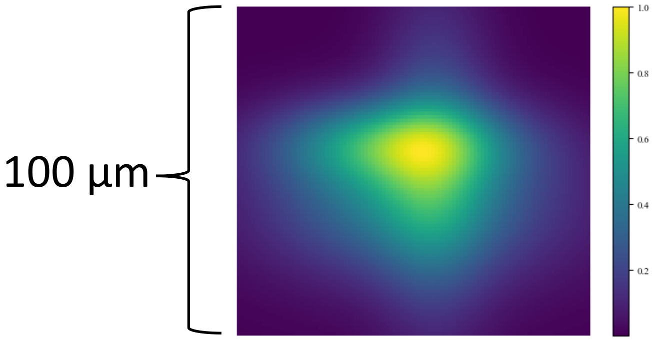
Reconstruction of the e- beam: “Measured COTR”

Set M=10, θ_m=0.28, γ=391(200MeV);

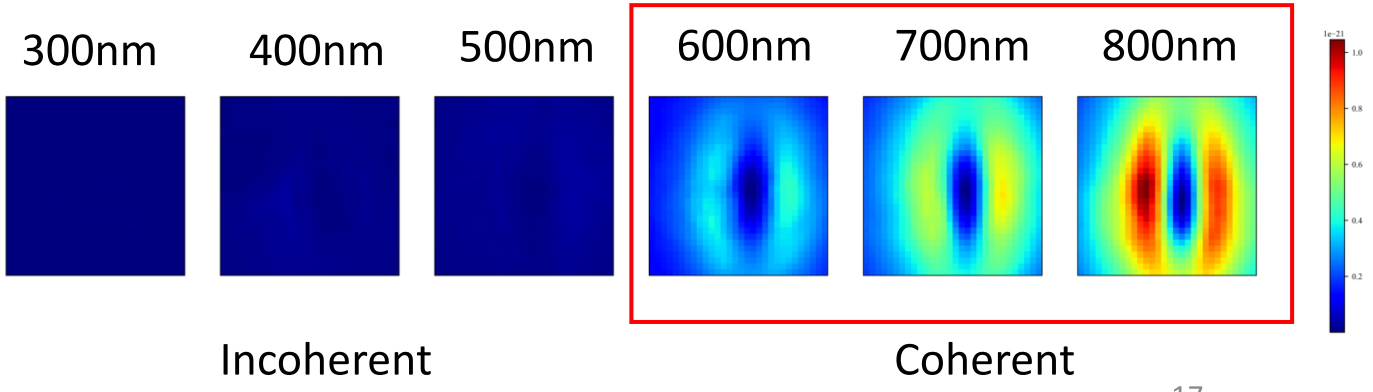
Set e- bunch: $\rho(x_s, y_s, z_s) = \sum_{i=1}^{4} N_{e_i} \frac{1}{\sqrt{2\pi}\sigma_{x_i}} \exp\left(-\frac{\left(x_i - \mu_{x_i}\right)^2}{2\sigma_{x_i}^2}\right) \frac{1}{\sqrt{2\pi}\sigma_{y_i}} \exp\left(-\frac{\left(y_i - \mu_{y_i}\right)^2}{2\sigma_{y_i}^2}\right) \frac{1}{\sqrt{2\pi}\sigma_{z_i}} \exp\left(-\frac{\left(z_i - \mu_{z_i}\right)^2}{2\sigma_{z_i}^2}\right)$

Params	ρ_1	ρ_2	ρ_3	ρ_4
N_e	1e9	0.7e9	0.5e9	1.5e9
μ_x	3μm	-7μm	-12μm	9μm
σ_x	34μm	15μm	18μm	9μm
μ_y	6μm	-3μm	4μm	-4μm
σ_y	11μm	25μm	34μm	23μm
μ_z	0.12μm	0.63μm	0.78μm	0.29μm
σ_z	0.15μm	0.25μm	0.35μm	0.4μm





e- bunch transverse profile



Reconstruction of the e- beam: Nonlinear least square fitting

1. **Forget e- bunch info in last page**
2. Set M=10, θ_m=0.28, γ=391(200MeV)
3. Presume e- bunch: $\rho(x_s, y_s, z_s) = \sum_{i=1}^{6} N_{e_i} \frac{1}{\sqrt{2\pi}\sigma_{x_i}} \exp\left(-\frac{\left(x_i-\mu_{x_i}\right)^2}{2\sigma_{x_i}^2}\right) \frac{1}{\sqrt{2\pi}\sigma_{y_i}} \exp\left(-\frac{\left(y_i-\mu_{y_i}\right)^2}{2\sigma_{y_i}^2}\right) \frac{1}{\sqrt{2\pi}\sigma_{z_i}} \exp\left(-\frac{\left(z_i-\mu_{z_i}\right)^2}{2\sigma_{z_i}^2}\right)$
4. Randomly set these 42 parameters, then generate $COTR_{\text{fitted}}$ at λ=600nm, 700nm, and 800nm
5. To minimize the cost function or objective function[1]:

$$\Phi(x, y) = \frac{1}{2} \| COTR_{\text{measured}}(x, y) - COTR_{\text{fitted}}(x, y) \|^2 \cdot W(x, y)$$

The minimization will stop when

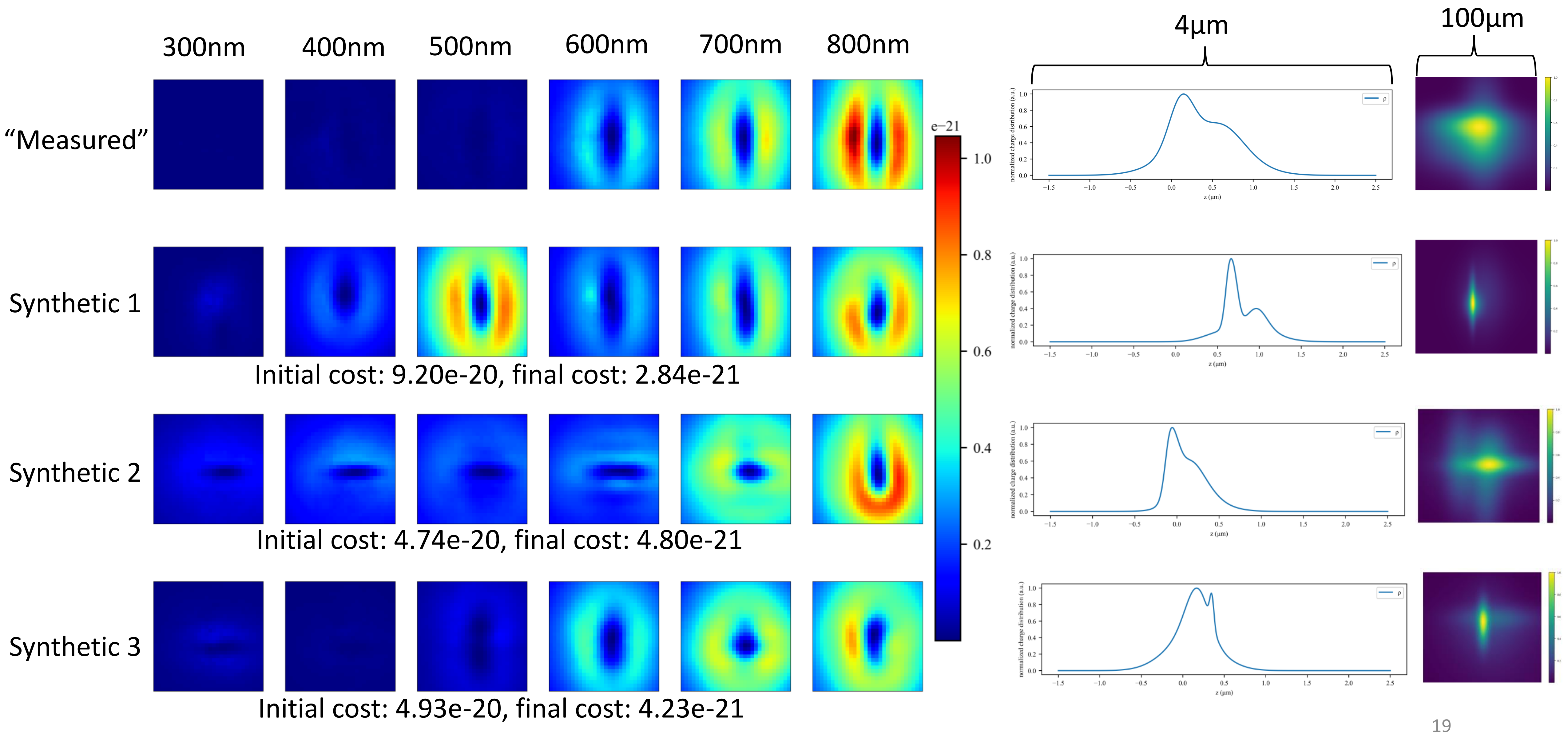
1) A minimum has been found within the user-defined precision (10^{-8}), OR

2) A user-defined maximum number of iteration has been reached (50)

Params	ρ_i
N_e	(0.5e9, 1e9)
μ_x	(-20μm, 20μm)
σ_x	(1μm, 30μm)
μ_y	(-20μm, 20μm)
σ_y	(1μm, 30μm)
μ_z	(0, 400μm)
σ_z	(0.1μm, 0.3μm)

[1]https://www.gnu.org/software/gsl/doc/html/nls.html#overview

Reconstruction of the e- beam: Synthetic COTR images



Reconstruction of the e- beam: Synthetic COTR images

What if the longitudinal or transverse profile is known?