Demonstration on 1D reconstruction of the electron beam by transition radiation

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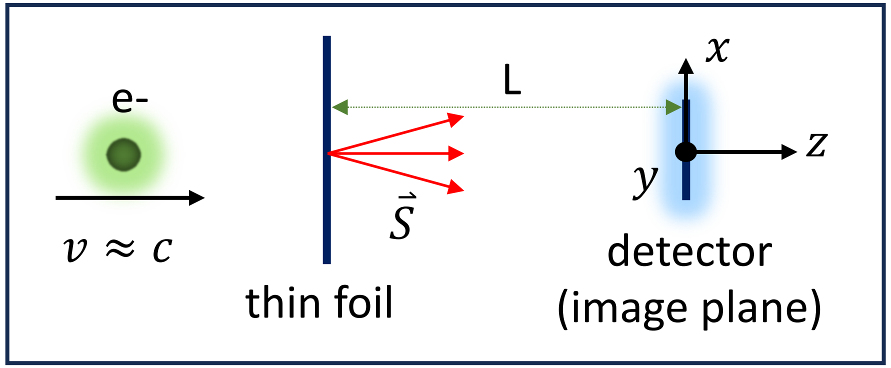
Apr 29th, 2024

1 Xiang et al. *NIM-A*, **570**, 3 (2007)

Content

TR angular distribution from a single electron (direct imaging)

Model (not in scale):



Angular (Ω) distribution (SI unit)[1]:

$$\frac{\mathrm{d}^2 W_e}{\mathrm{d}\omega \mathrm{d}\Omega} = \frac{r_e m_e c}{\pi^2} \frac{\beta^2 \sin^2\theta}{(1-\beta^2\cos^2\theta)^2}$$

(ω-independent)

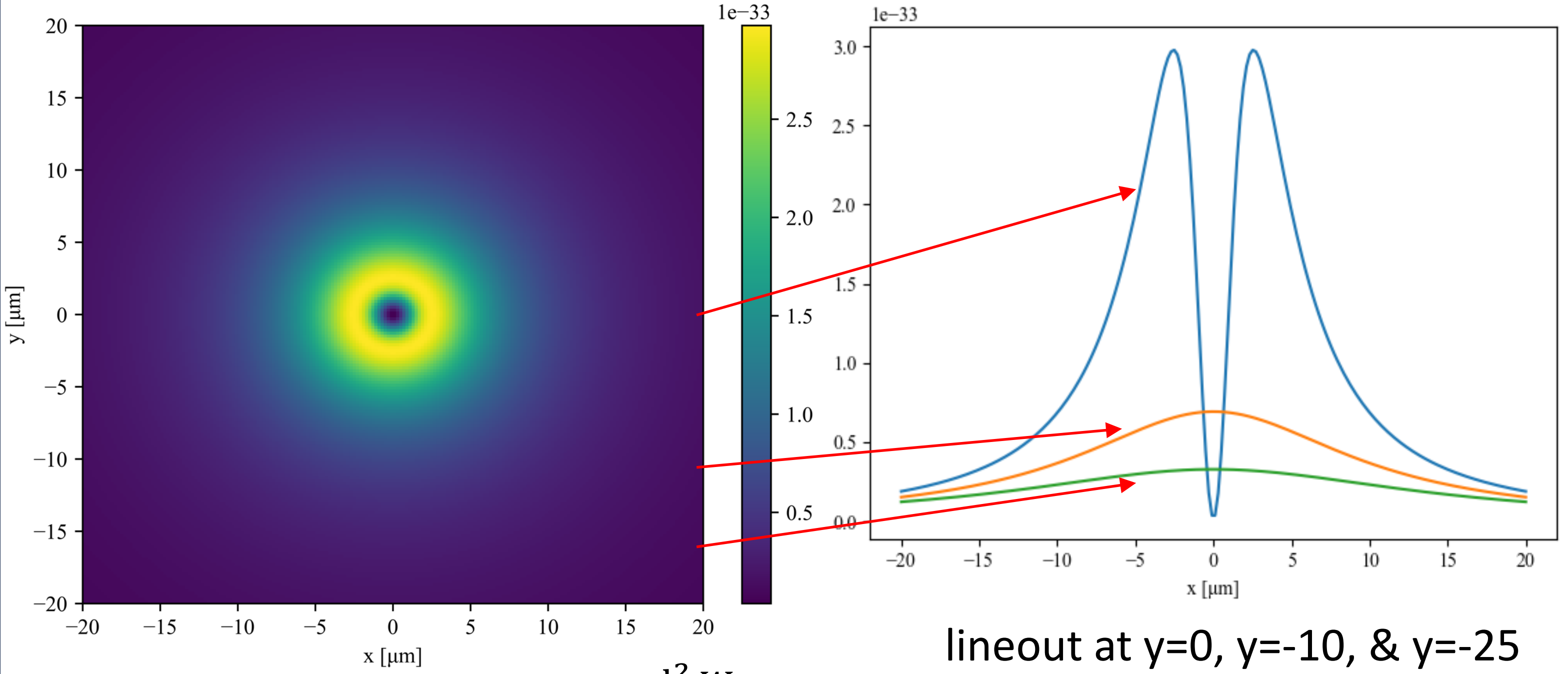
W_e: power (W)

θ: spanned by $\vec{S}$ and z axis

r_e: classical electron radius

Set: L=1 mm, γ= 391 (200 MeV)

Result: x and y stands for the image plane



(power angular distribution $\frac{\mathrm{d}^2 W_e}{\mathrm{d}\omega \mathrm{d}\Omega}$)

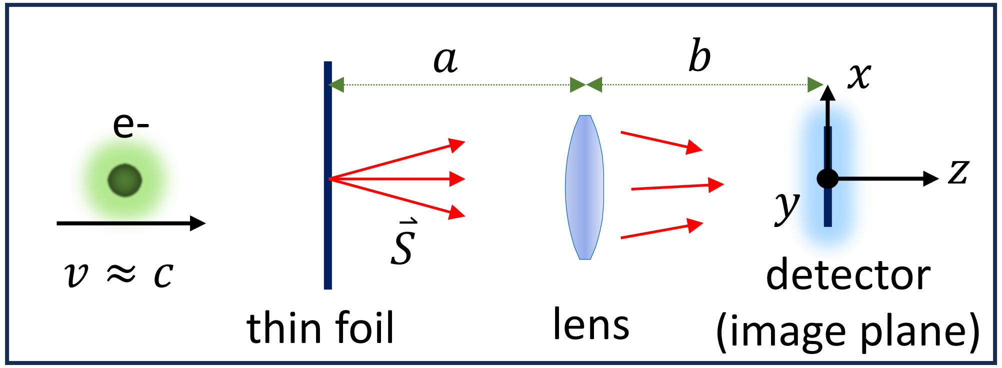
lineout at y=0, y=-10, & y=-25

Peak at $\theta = \frac{1}{\gamma}$ (pivotal character)

1 Schroeder et al. *PRE* **69**, 016501 (2004)

TR angular distribution from a single electron (focused by lens)

Model (not in scale):



$\vec{E}$ field on the image plane (CGS unit)[1]:

$$\vec{E}(x,y) = \frac{2e}{\lambda M v} f(\theta_m, \gamma, \zeta)\vec{e}_r$$

where $f(\theta_m, \gamma, \zeta) = \int_0^{\theta_m} \frac{\theta^2}{\theta^2+\gamma^{-2}} J_1(\zeta\theta)\mathrm{d}\theta$, $\zeta = \frac{kR}{M}$, $M = \frac{b}{a}$;

θ_m is the acceptance angle of the lens (or N.A.);

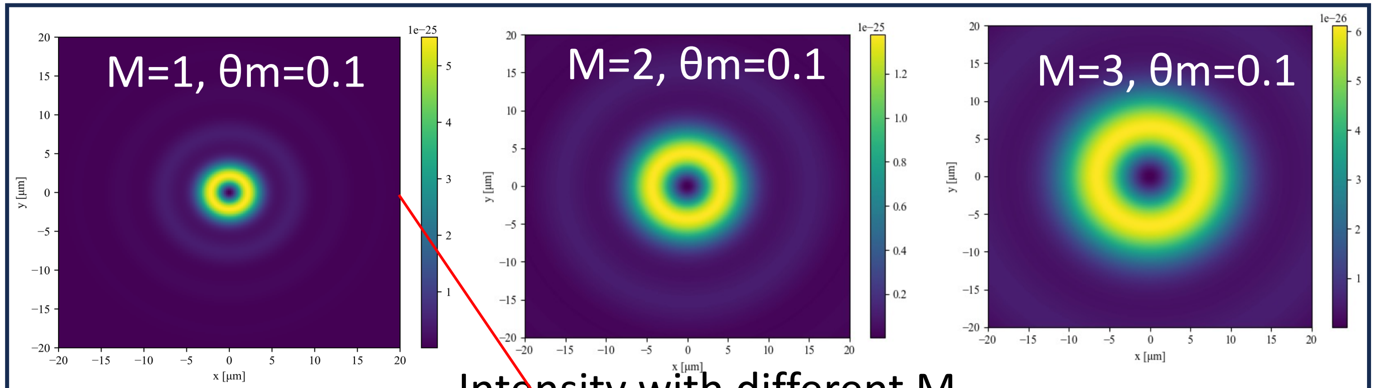
The intensity spectral density is $I_\omega(x,y) = \frac{c}{4\pi^2}\left|\vec{E}(x,y)\right|^2$,

also known as Point Spread Function (PSF) in the frequency domain

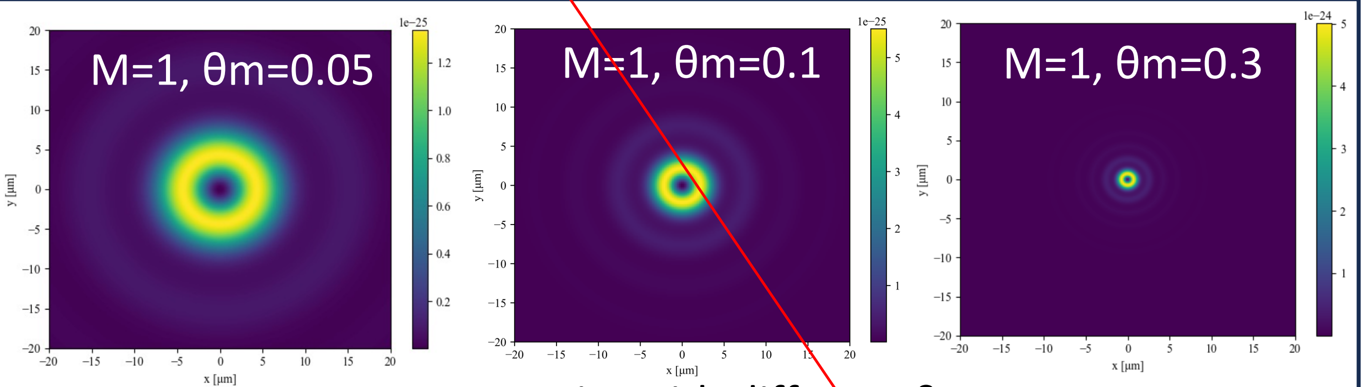
1 Xiang et al. *Nucl. Instrum. Meth. A* **570**, 3 (2007)

Set: λ=500nm, γ= 391 (200 MeV), a+b=L

Result: x and y stands for the image plane

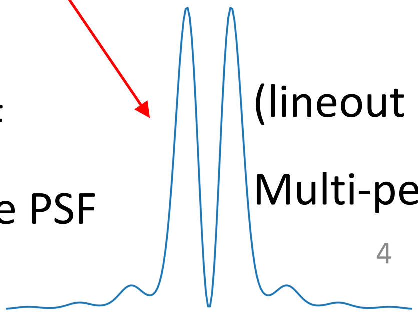


Intensity with different M



Intensity with different θm

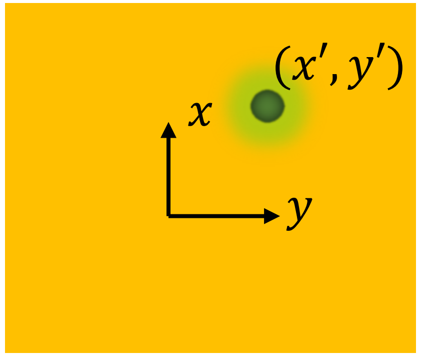
1. The bigger the M, the wider the PSF
2. The bigger the θm, the narrower the PSF



(lineout at y=0)

Multi-peak

TR angular distribution from multiple electrons (focused by lens)



foil plane

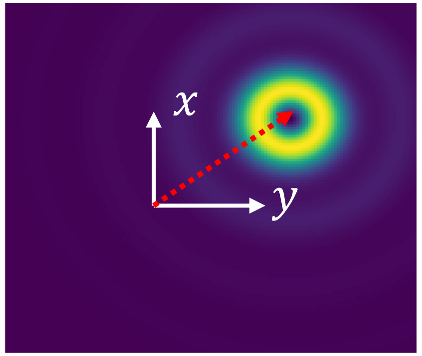


image plane

(foil plane & image plane have the same size.)

- Give e- distribution: $\sigma(x, y)$;
- The electrons at point (x', y') will generate TR in the image plane with intensity spectral $I'_\omega(x, y) \propto I_\omega(x - x', y - y')$, which is a translation from the origin
- The proportional coefficient is

$$k = \frac{\sigma(x',y')\mathrm{d}x'\mathrm{d}y'}{e} = \frac{\sigma(x',y')\mathrm{d}x'\mathrm{d}y'}{e}$$

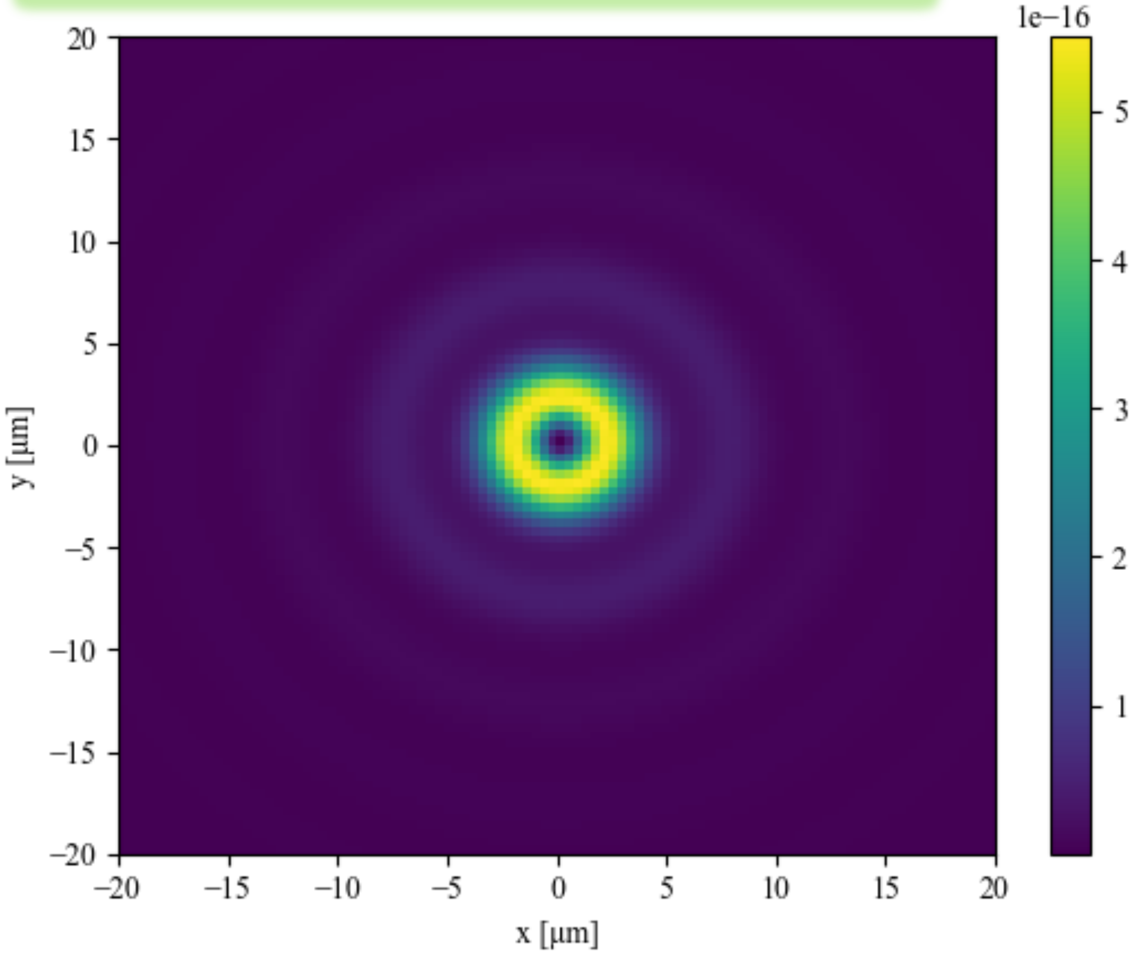
- Integral over all points on the foil plane will lead to the total intensity spectral (assume added incoherently)
- Final intensity spectral is a convolution between e- distribution and PSF:

$$I_{\Sigma,\omega}(x, y) = \frac{1}{e}\iint_{-\infty}^{+\infty} \sigma(x', y') I_\omega(x - x', y' - y)\mathrm{d}x'\mathrm{d}y'$$

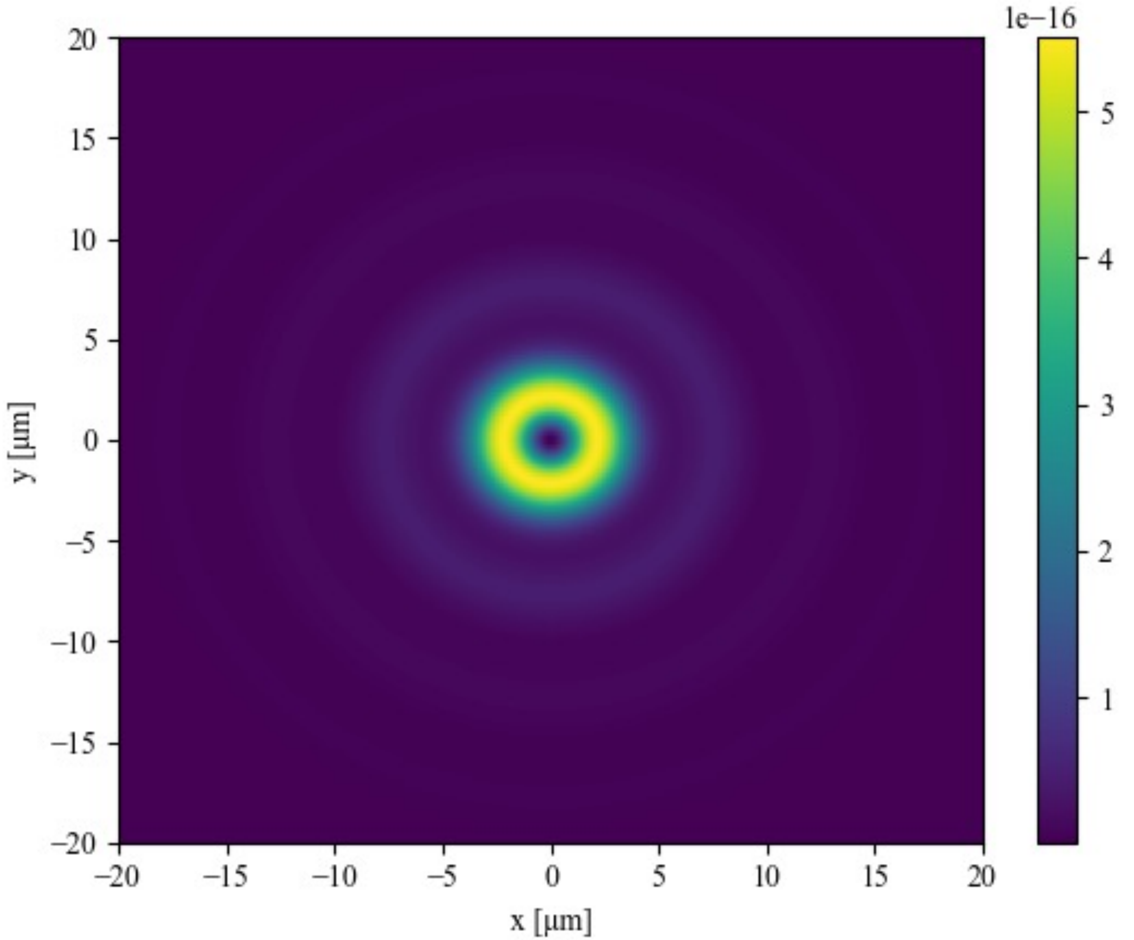
(Manifestation: Setting $\sigma(x', y') = e\delta^2(x', y')$= $e\delta^2(0,0)$ will reduce to the situation of a single electron)

TR angular distribution from multiple electrons (focused by lens)

Situation 1: **sharp**-2D-Gaussian distribution



TR from sharp Gaussian Distribution



e- accumulate at the origin (identical charges)

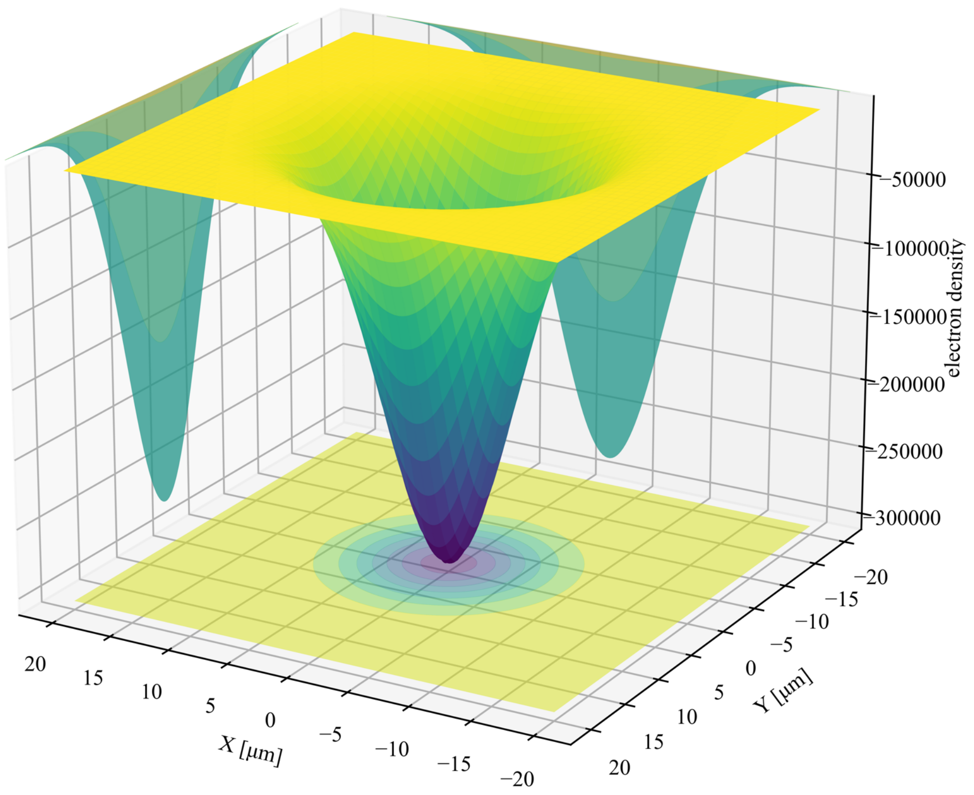
⇒Two figures are quite similar despite the grid differences.

⇒Proved the correctness of computation code and incoherent TR theory.

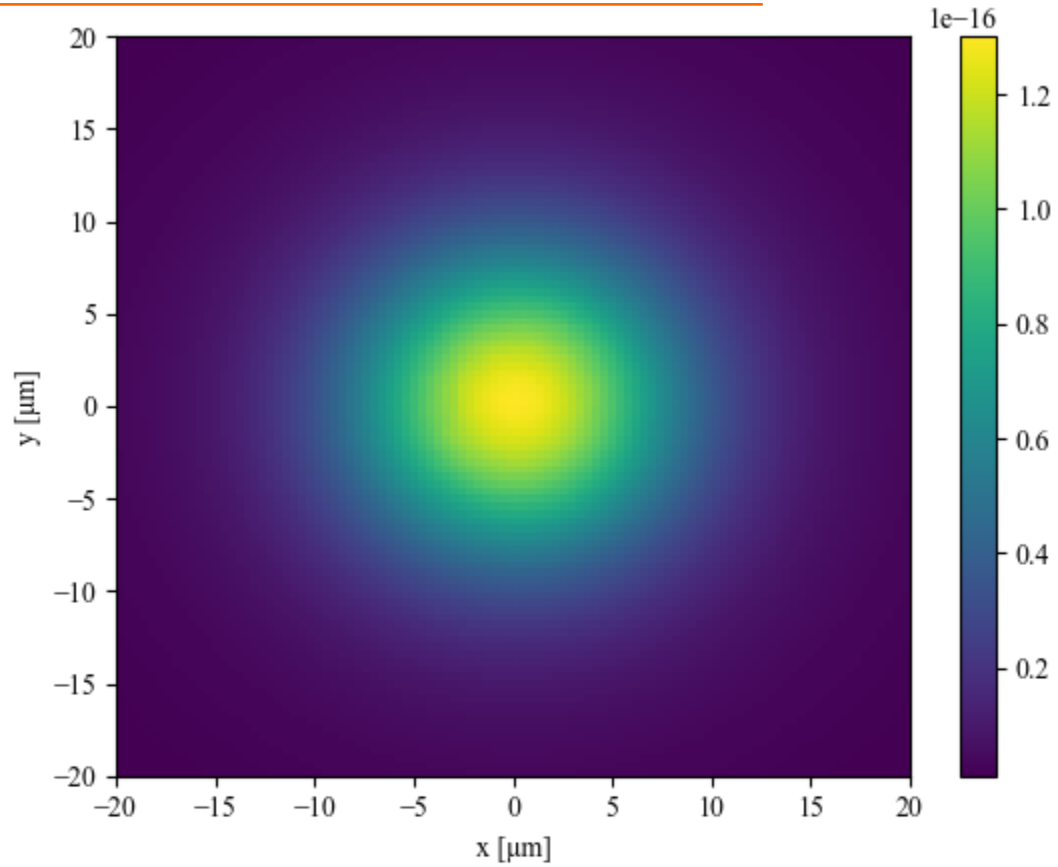
TR angular distribution from multiple electrons (focused by lens)

Situation 2: **moderate** 2D Gaussian distribution

$$\sigma(x,y) = \frac{Nq}{2\pi\sigma_x\sigma_y}\exp\left(-\left(\frac{x^2}{2\sigma_x^2}+\frac{y^2}{2\sigma_y^2}\right)\right)$$



Set N=1e9; $\sigma_x = \sigma_y$=5 μm



TR from moderate Gaussian Distribution

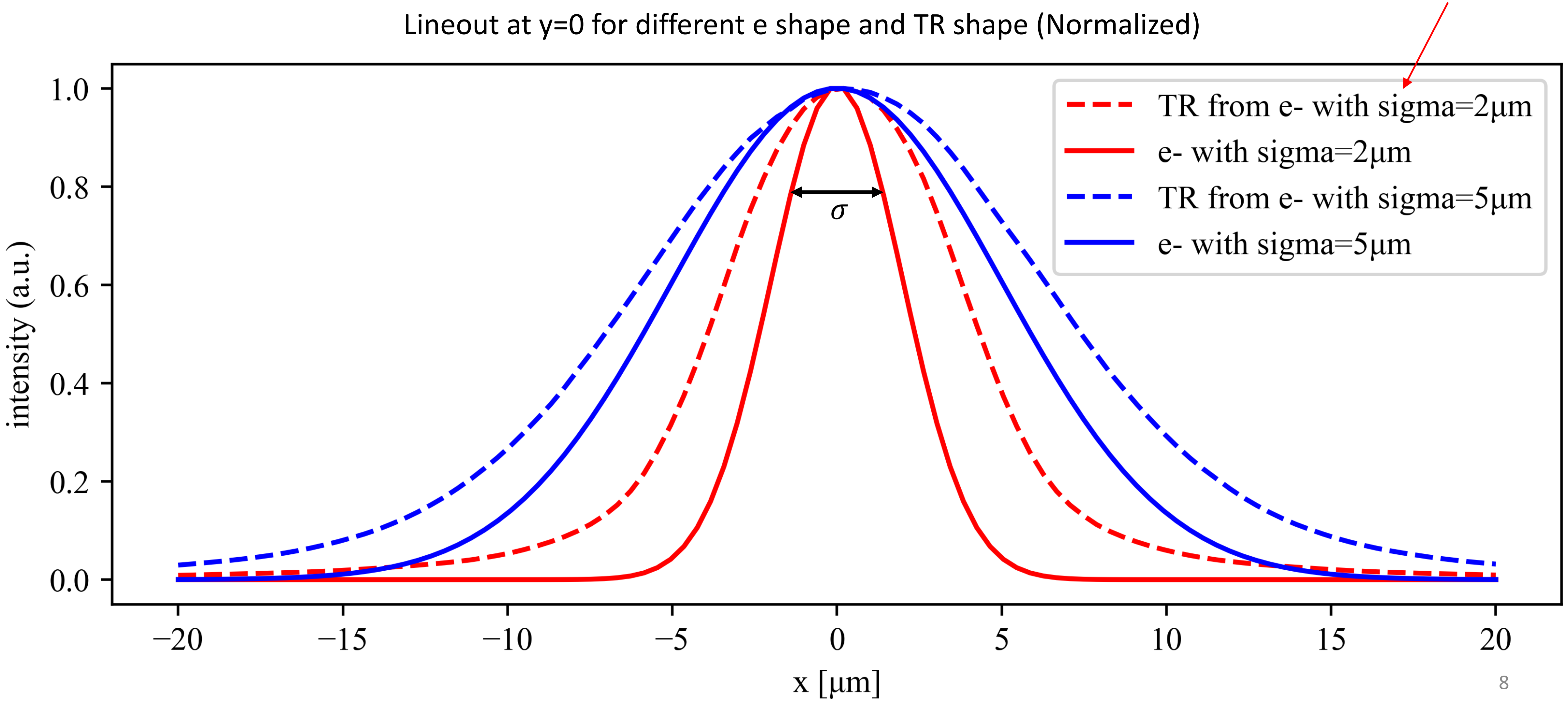
⇒ TR has a Gaussian-like shape **as well**.

(**Correlation** between the e- shape and TR shape?)

TR angular distribution from multiple electrons (focused by lens)

Situation 2: **moderate** 2D Gaussian distribution

FWHM=2.355*sigma



e- reconstruction from TR images

- If knowing e- distribution, TR image is physically given by a convolution (already shown before):

$$I_{\Sigma,\omega}(x,y)=\frac{1}{e}\iint_{-\infty}^{+\infty}\sigma(x',y')I_{\omega}(x-x',y'-y)\mathrm{d}x'\mathrm{d}y'$$

- **Inverse question**: If knowing TR image, how to deduce the e-distribution?
  - Deconvolution? (remain studied)
  - Parameter optimization (**demonstrated below**)

$$I_{\Sigma,\omega}(x,y)=\frac{1}{e}\iint_{-\infty}^{+\infty}\sigma(x',y')I_{\omega}(x-x',y'-y)\mathrm{d}x'\mathrm{d}y'$$

- $I_{\Sigma,\omega}(x,y)$: TR images
- $\sigma(x',y')$: e- distribution (trying to reconstruct)
- $I_{\omega}(x-x',y'-y)$: convolution kernel

Main idea: the e- shape is pre-set to a known function with **parameters**

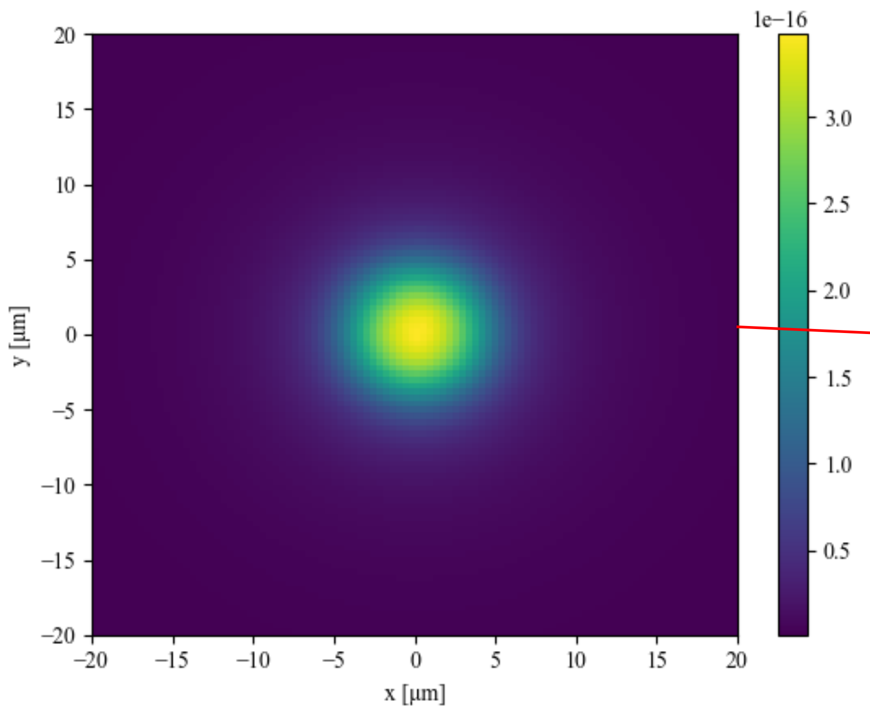
e- reconstruction from TR images (parameter optimization)

Step 1: Generate TR

$$\sigma(x,y) = \frac{Nq}{2\pi\sigma_x\sigma_y}\exp\left(-\left(\frac{x^2}{2\sigma_x^2}+\frac{y^2}{2\sigma_y^2}\right)\right)$$

(Set N=1e9, $\sigma_x = \sigma_y$=2 μm)

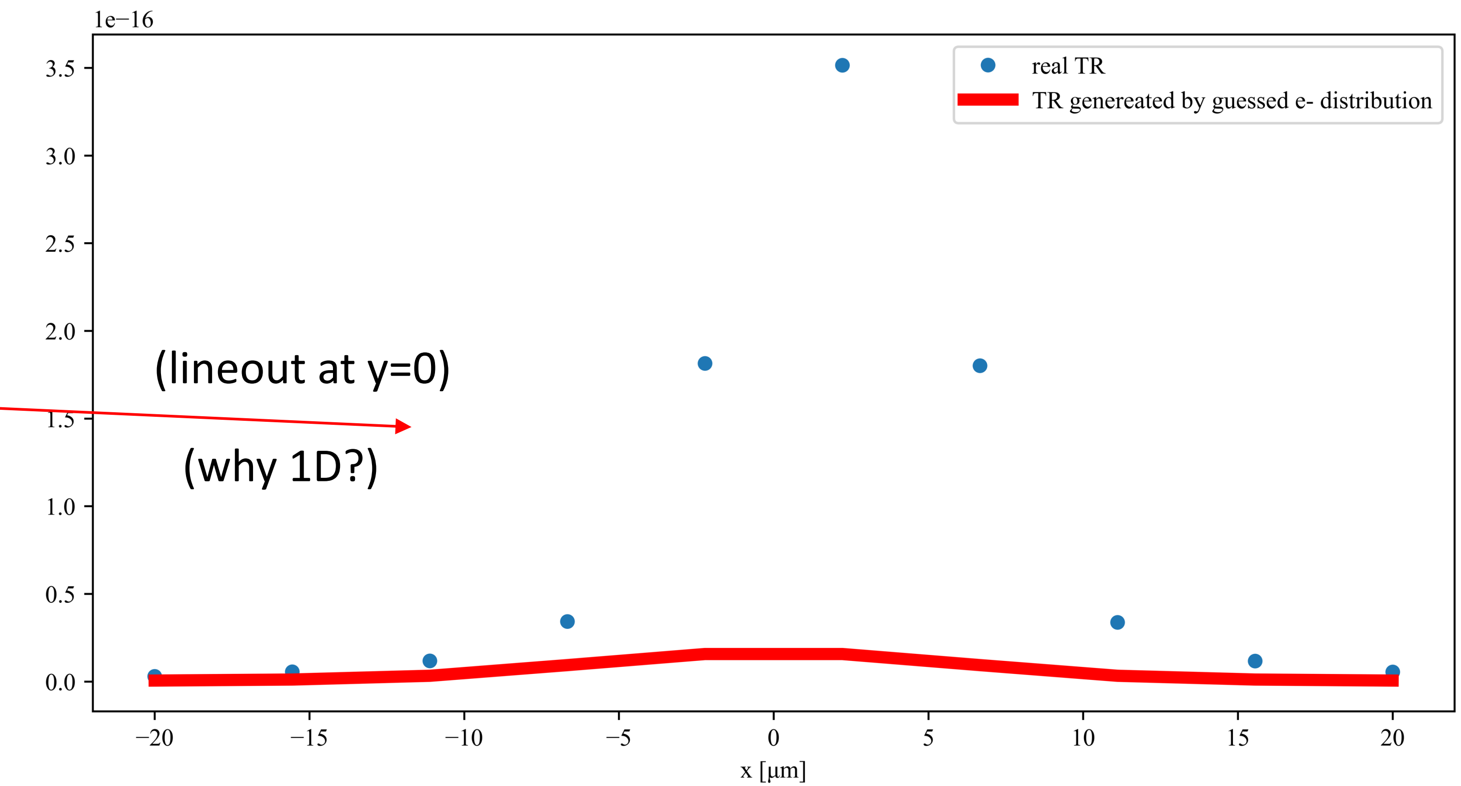
reconstruct later!



Step 2: with the known TR, try to guess an e- distribution, here would be Gaussian

(Set N=1.1e9, σ_x=5.1 μm, σ_y=50 μm)

Step 3: with the guessed e- distribution, one can compare the so generated TR with the known TR.



e- reconstruction from TR images (parameter optimization)

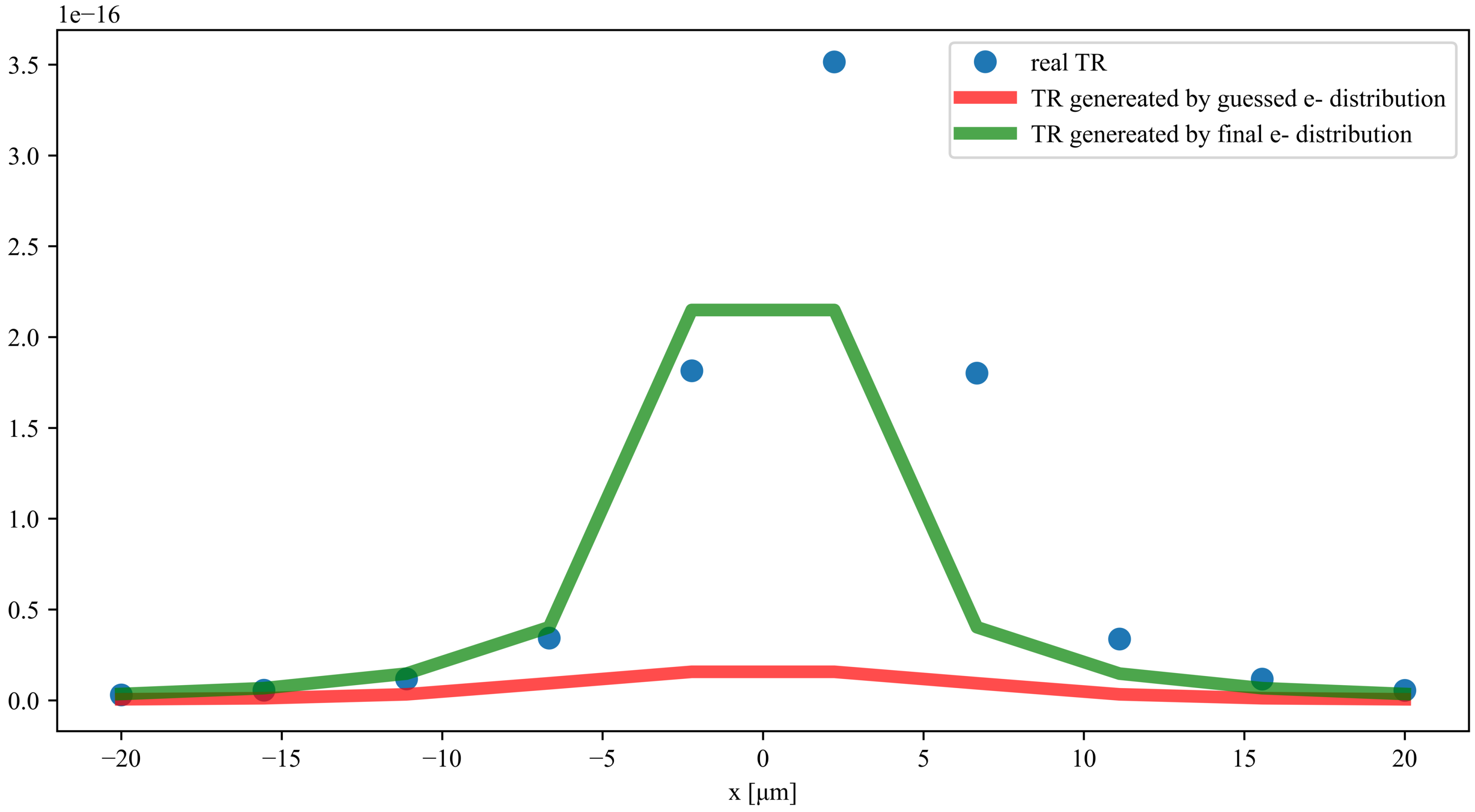
Step 4: Use optimization method to improve the parameters of e- distribution, by comparing the output TR with the known TR. Say, to minimize the following

$$\sum_{x_i,y_i} \left\| \frac{TR_{known}(x_i,y_i) - TR_{generated}(x_i,y_i)}{TR_{known}(x_i,y_i)} \right\|^2$$

Converged parameters:

(N=2.09e9, σ_x=1.3 μm, σ_y=10.5 μm)

, which leads to the 1D TR image lining out at y=0



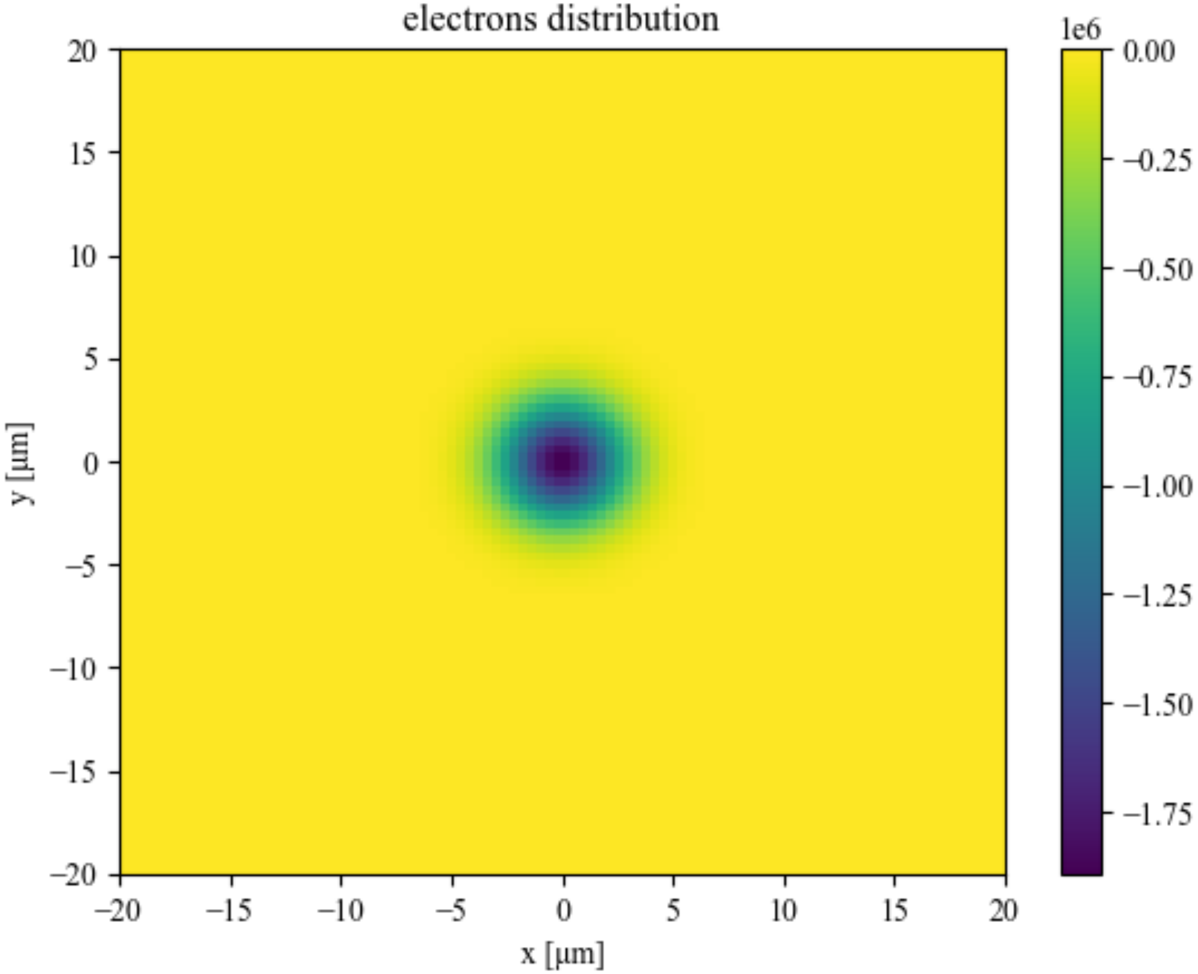
Due to time-consuming optimization logarithm, only choose

1. Few points
2. 1D rather than 2D

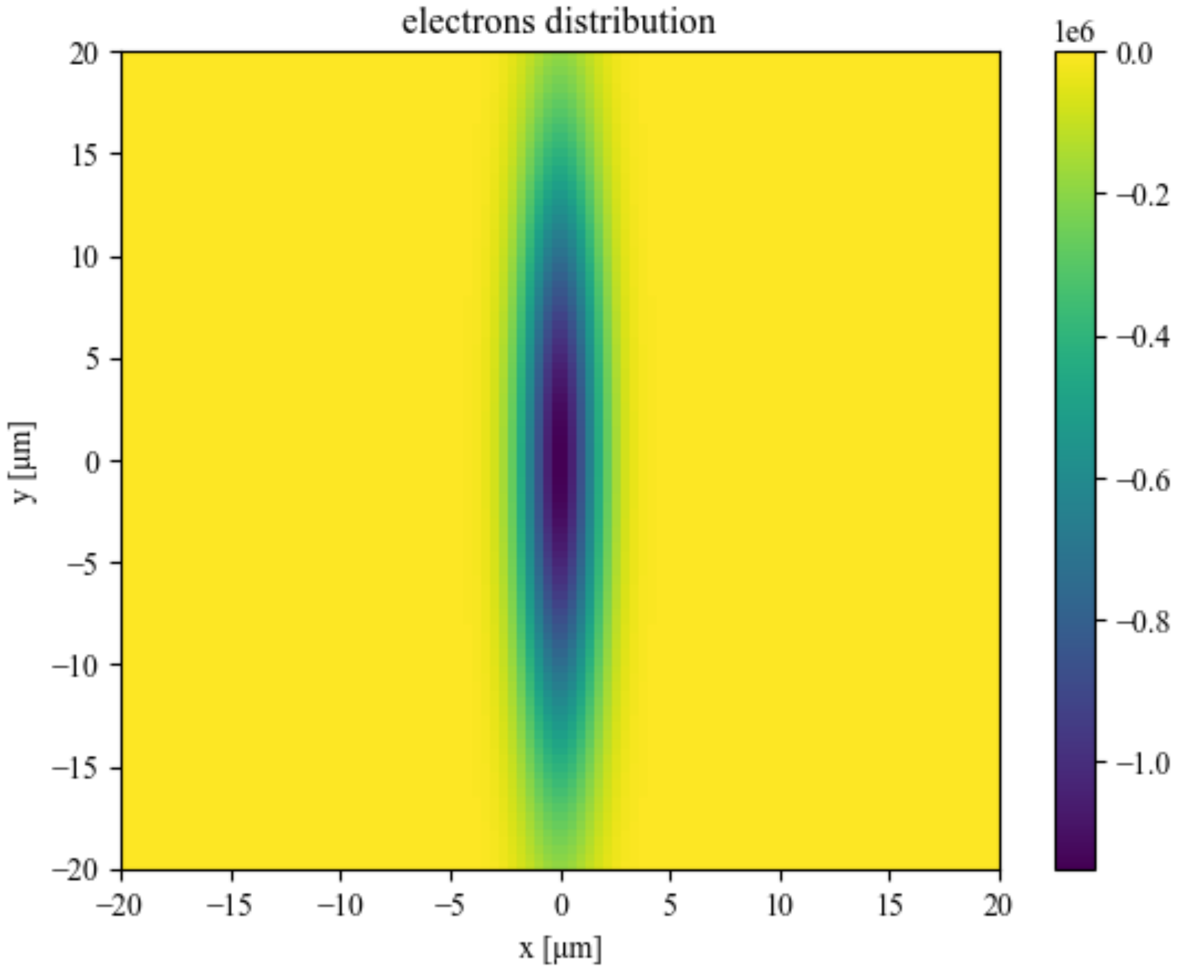
e- reconstruction from TR images (parameter optimization)

Step 5: comparison (e- distribution)

$$\sigma(x,y) = \frac{Nq}{2\pi\sigma_x\sigma_y}\exp\left(-\left(\frac{x^2}{2\sigma_x^2}+\frac{y^2}{2\sigma_y^2}\right)\right)$$



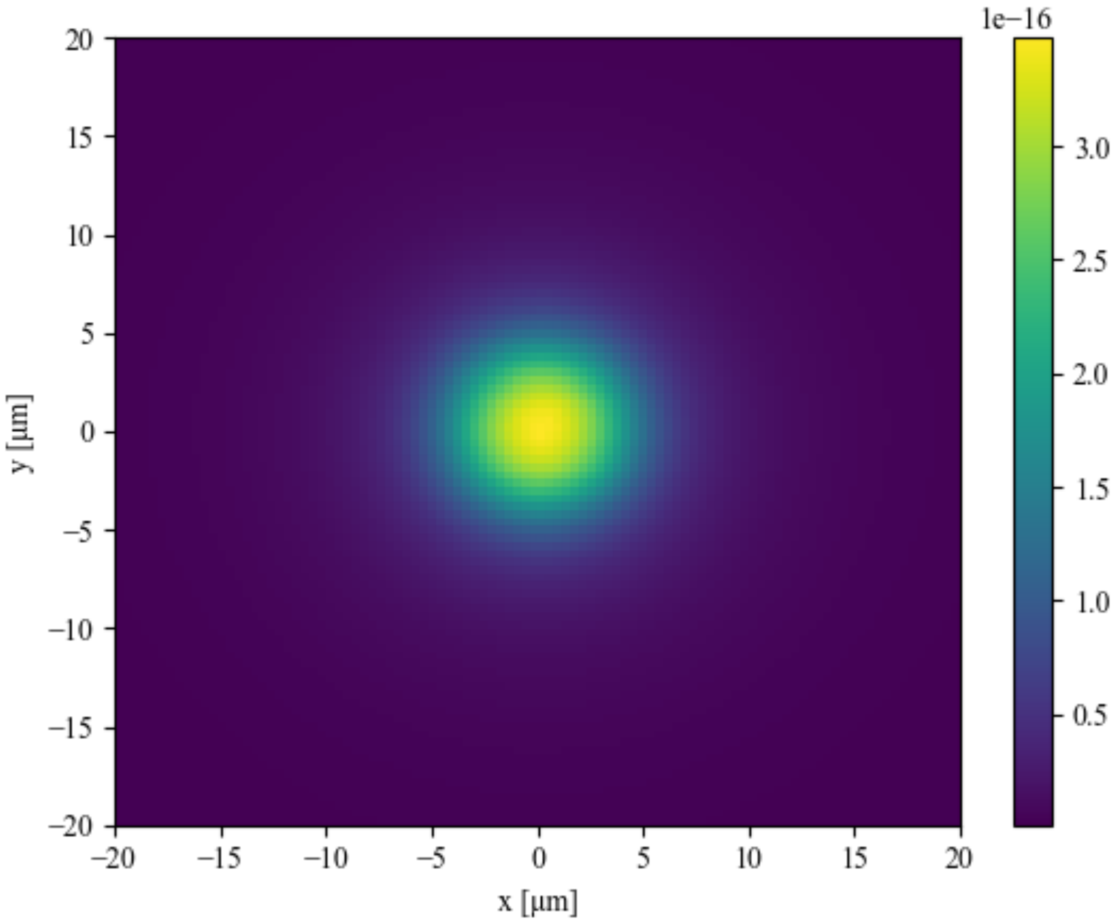
actual e- distribution
(N=1e9, σ_x= σ_y=2 µm)



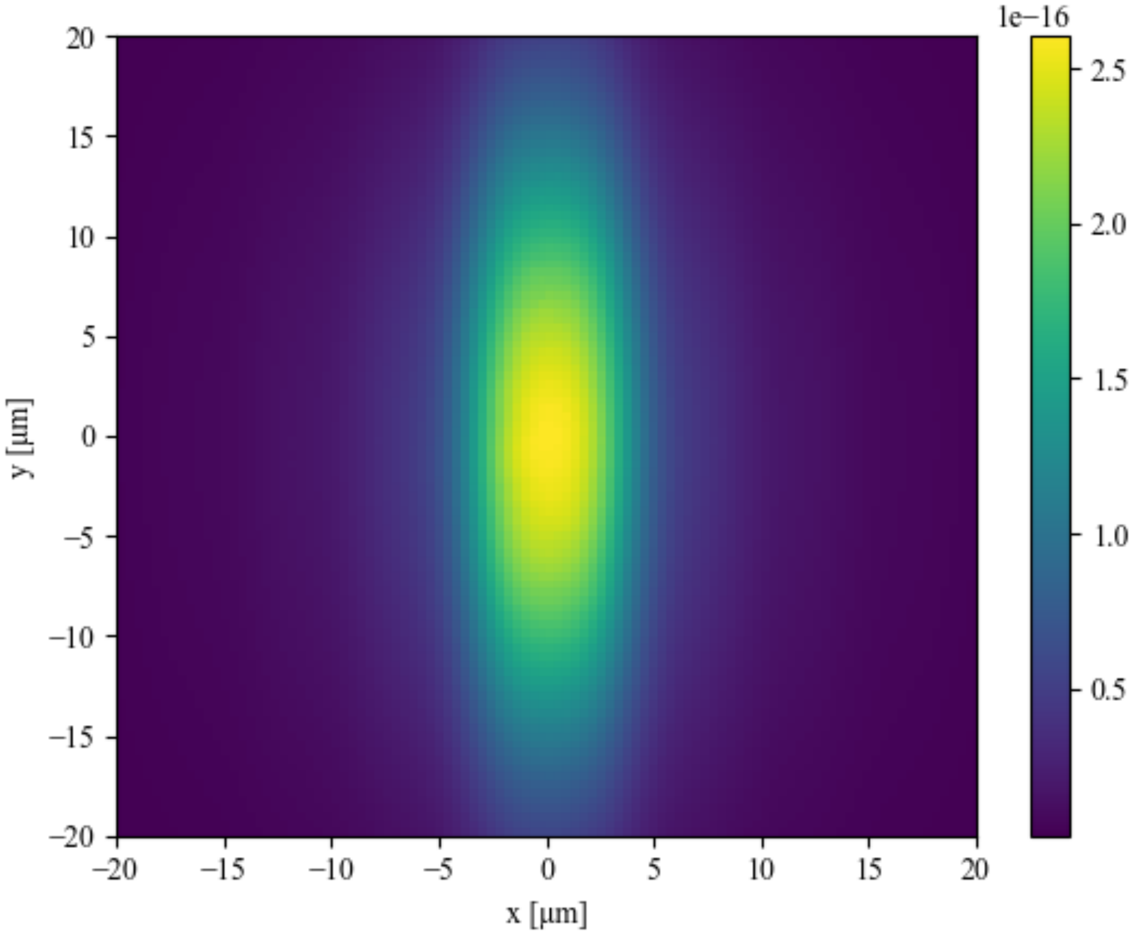
reconstructed e- distribution
(N=2.09e9, σ_x=1.3 µm, σ_y=10.5 µm)

e- reconstruction from TR images (parameter optimization)

Step 6: comparison (TR)



Actual TR



TR from reconstructed e- distribution

Recap and further discussion

Recap:

1. Calculated TR from a single electron & bunch electrons, both in the far field (direct imaging) and near field situation (focused by lens).
2. Tried to reconstruct e- distribution in one dimension, based on an assumption that e- distribution is a form-known, parameter-unknown function
3. The one-dimension reconstruction **seems** to work.
4. All the calculations shown are done by self-written C++ and Python code.

Further discussion:

1. Ways to improve the speed of parameter optimization methods (C++, parallel computing, GPU computing by CUDA, HPC, …)
2. How to extend to the z axis?

⇒ Consider the coherent TR

⇒ x, y, and z axis distribution lead to phase difference (the criterion of coherence; N^2 makes a big difference)

$$\mathbf{E}_{\perp}^{(n)}(k,x,y) = Q\int dx'\int dy'\mathbf{E}_{\perp}^{(\mathrm{PSF})}(\mathbf{r}-\mathbf{r}')\int e^{-ikz}\rho_n(x',y',z)dz$$