轴子电动力学中的基本方程

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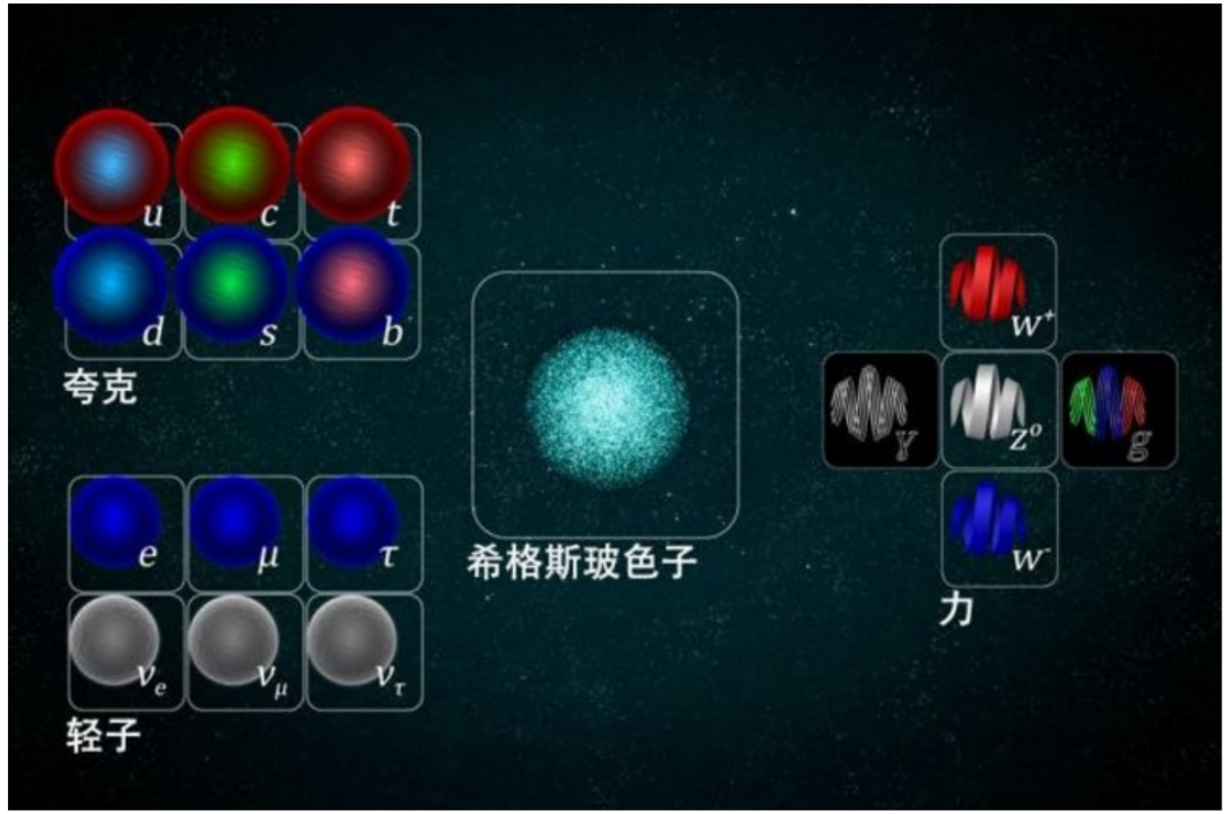
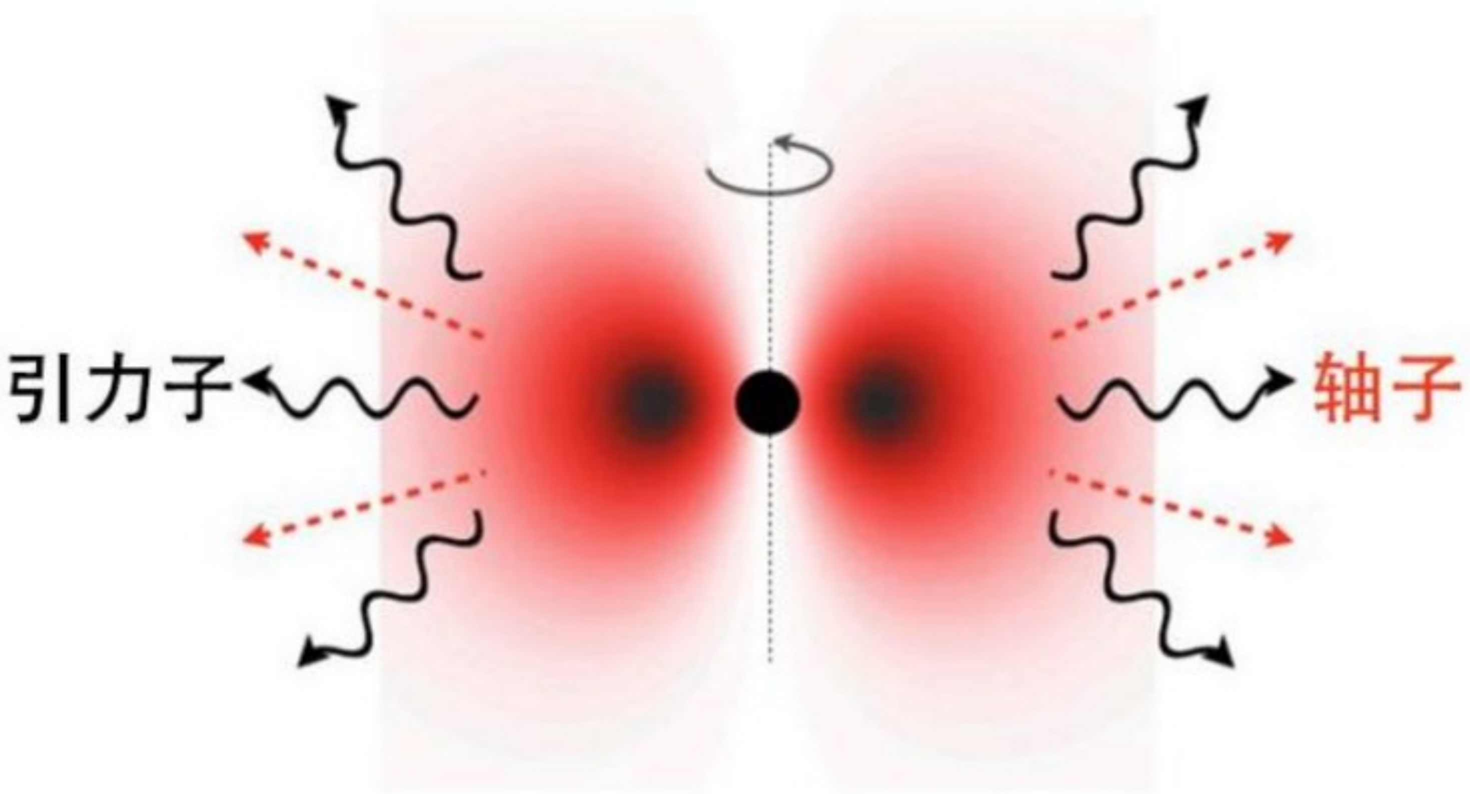


轴子简介

轴子是自旋为0的中性玻色子，是为了解决量子色动力学中的强CP问题而引入的**假想粒子**。同时，由于轴子与标准模型中的粒子之间的相互作用极弱，其又成为解释暗物质的理想候选者。



经典电磁场的数学描述——麦克斯韦方程组

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \times \vec{B} = \mu_0 \vec{j} + \varepsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t}$$

$$\nabla \cdot \vec{B} = 0$$

存在电介质和磁介质

定义：$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \cdot \vec{D} = \rho_0$$

$$\nabla \times \vec{H} = \vec{j}_0 + \frac{\partial \vec{D}}{\partial t}$$

$$\nabla \cdot \vec{B} = 0$$

已经考虑到极化电荷和磁化电流

轴子？

轴子电动力学方程的推导

写出耦合了轴子作用的拉格朗日量:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu a)^2 - \frac{1}{2}m_a^2 a^2 - \frac{1}{4}F_{\mu\upsilon}F^{\mu\upsilon} + \frac{1}{4}g_{a\gamma\gamma}aF_{\mu\upsilon}\widetilde{F}^{\mu\upsilon}$$

结合场论中的方法,可以推出:

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \cdot \vec{D} = \rho_0 + g_{a\gamma\gamma}\sqrt{\frac{\varepsilon_0}{\mu_0}}\vec{B} \cdot \nabla a$$

$$\nabla \times \vec{H} = \vec{j}_0 + \frac{\partial \vec{D}}{\partial t} - g_{a\gamma\gamma}\sqrt{\frac{\varepsilon_0}{\mu_0}}(\vec{B}\frac{\partial a}{\partial t} + \nabla a \times \vec{E})$$

$$\nabla \cdot \vec{B} = 0$$

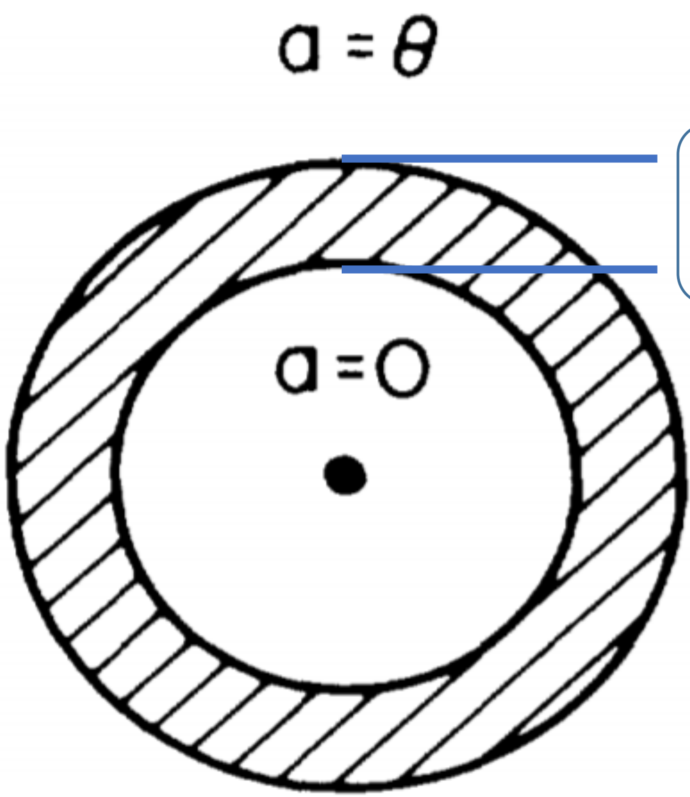
a代表轴子标量场,即

$$a = a(t, \vec{r}) = a_0 cos(\vec{k}_a \cdot \vec{r} - \omega_a t)$$

波长远远大于实验室尺度，故可以忽略某点位矢的影响.

$$a \approx a(t) \approx a_0 cos(\omega_a t)$$

应用——Dyon Charge



一磁单极子被薄球壳包围，内部轴子标量场为$a = 0$, 外部为$a = \theta$.

$$\nabla a = \frac{\partial a}{\partial r}\vec{e}_r = \theta \vec{e}_r$$

由 $\nabla \vec{D} = \rho_0 + g_{a\gamma\gamma}\sqrt{\frac{\varepsilon_0}{\mu_0}}\vec{B}\nabla a$

积分可得等效电荷 $Q = \Phi \theta g_{a\gamma\gamma}\sqrt{\frac{1}{\varepsilon_0 \mu_0}}$

在轴子的作用下，电生磁、磁生电有了新的方式

轴子电动力学方程的修改

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \cdot \vec{D} = \rho_0 + g_{a\gamma\gamma}\sqrt{\frac{\varepsilon_0}{\mu_0}}\vec{B} \cdot \nabla a$$

$$\nabla \times \vec{H} = \vec{j}_0 + \frac{\partial \vec{D}}{\partial t} - g_{a\gamma\gamma}\sqrt{\frac{\varepsilon_0}{\mu_0}}(\vec{B}\frac{\partial a}{\partial t} + \nabla a \times \vec{E})$$

$$\nabla \cdot \vec{B} = 0$$

1. 没有赋予在轴子参与下，新项的物理含义
2. $\vec{D}$, $\vec{H}$描述轴子电动力学不合适
3. 形式复杂

对上述方程的修改（使方程不显含轴子影响项）

寻找可以替代$\vec{D}, \vec{H}$的物理量

$$\nabla \cdot \vec{D} = \rho_0 + g_{a\gamma\gamma}\sqrt{\frac{\varepsilon_0}{\mu_0}}\vec{B}\cdot\nabla a$$

$$\nabla \times \vec{H} = \vec{j}_0 + \frac{\partial \vec{D}}{\partial t} - g_{a\gamma\gamma}\sqrt{\frac{\varepsilon_0}{\mu_0}}\left(\vec{B}\frac{\partial a}{\partial t} + \nabla a \times \vec{E}\right)$$

期望得到以下形式：

$$\nabla \cdot \vec{D}_T = \rho_0$$

$$\nabla \times \vec{H}_T = \vec{j}_0 + \frac{\partial \vec{D}_T}{\partial t}$$

式中：

$$\vec{D}_T = \vec{D} + \vec{P}_{aB}$$

$$\vec{H}_T = \vec{H} - \vec{M}_{aE}$$

对上述方程的修改($\vec{D}$)

$$\nabla\cdot\vec{D}=\rho_0+g_{a\gamma\gamma}\sqrt{\frac{\varepsilon_0}{\mu_0}}\vec{B}\cdot\nabla a$$

$$=\rho_0+g_{a\gamma\gamma}\sqrt{\frac{\varepsilon_0}{\mu_0}}[\nabla\cdot a\vec{B}+a(\nabla\cdot\vec{B})]$$

$$\nabla\cdot\vec{B}=0$$

$$=\rho_0+g_{a\gamma\gamma}\sqrt{\frac{\varepsilon_0}{\mu_0}}\nabla\cdot a\vec{B}$$

$$\Rightarrow\nabla\cdot\vec{D}-g_{a\gamma\gamma}\sqrt{\frac{\varepsilon_0}{\mu_0}}\nabla\cdot a\vec{B}=\rho_0$$

$$\Rightarrow\nabla\cdot[\vec{D}-g_{a\gamma\gamma}\sqrt{\frac{\varepsilon_0}{\mu_0}}\cdot a\vec{B}]=\rho_0$$

此即要寻找的新物理量 $\vec{D}_T$，其中 $\vec{P}_{aB}=-g_{a\gamma\gamma}\sqrt{\frac{\varepsilon_0}{\mu_0}}\cdot a\vec{B}$

对上述方程的修改 ($\vec{H}$)

$$\nabla\times\vec{H}=\vec{j}_0+\frac{\partial\vec{D}}{\partial t}-g_{a\gamma\gamma}\sqrt{\frac{\varepsilon_0}{\mu_0}}(\vec{B}\frac{\partial a}{\partial t}+\nabla a\times\vec{E})$$

注意到 $\nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}$

$$=\vec{j}_0+\frac{\partial\vec{D}}{\partial t}-g_{a\gamma\gamma}\sqrt{\frac{\varepsilon_0}{\mu_0}}[\vec{B}\frac{\partial a}{\partial t}+(\nabla\times a\vec{E})-a(\nabla\times\vec{E})]$$

$$\Rightarrow\nabla\times\vec{H}=\vec{j}_0+\frac{\partial\vec{D}}{\partial t}-g_{a\gamma\gamma}\sqrt{\frac{\varepsilon_0}{\mu_0}}[\vec{B}\frac{\partial a}{\partial t}+(\nabla\times a\vec{E})-a(-\frac{\partial\vec{B}}{\partial t})]$$

$$=\vec{j}_0+\frac{\partial\vec{D}}{\partial t}-g_{a\gamma\gamma}\sqrt{\frac{\varepsilon_0}{\mu_0}}[\frac{\partial(a\vec{B})}{\partial t}+(\nabla\times a\vec{E})]$$

考虑到：$\frac{\partial D_T}{\partial t}=\frac{\partial D}{\partial t}-g_{a\gamma\gamma}\sqrt{\frac{\varepsilon_0}{\mu_0}}\cdot\frac{\partial(a\vec{B})}{\partial t}$

代入并移项： $\nabla\times[\vec{H}+g_{a\gamma\gamma}\sqrt{\frac{\varepsilon_0}{\mu_0}}(a\vec{B})]=\vec{j}_0+\frac{\partial\vec{D}_T}{\partial t}$

此即要寻找的新物理量 :$\vec{H}_T$ ，其中 $\vec{M}_{aB}=-g_{a\gamma\gamma}\sqrt{\frac{\varepsilon_0}{\mu_0}}(a\vec{E})$

对上述方程的修改($\vec{D}$,$\vec{H}$)

将修改后的$\vec{D}$和$\vec{H}$代入麦克斯韦方程组,可得：

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \cdot \vec{D}_T = \rho_0$$

$$\nabla \times \vec{H}_T - \frac{\partial \vec{D}_T}{\partial t} = \vec{j}_0$$

$$\nabla \cdot \vec{B} = 0$$

下面的工作，将进一步略去$\vec{E}$和$\vec{B}$

对上述方程的修改$(\vec{E},\vec{B})$

仿照经典电磁理论中$\vec{E}$和$\vec{D}$, $\vec{B}$和$\vec{H}$的关系,有:

总电场强度 $\vec{E}_T = \frac{1}{\varepsilon}\vec{D}_T = \frac{1}{\varepsilon_r\varepsilon_0}\vec{D}_T$

总磁感应强度 $\vec{B}_T = \mu\vec{H}_T = \mu_r\mu_0\vec{H}_T$

可以得到： $\vec{E}_T = \vec{E} + \vec{E}_{aB}$,式中 $\vec{E}_{aB} = -g_{a\gamma\gamma}\frac{c}{\varepsilon_r}(a\vec{B})$

$\vec{B}_T = \vec{B} + \vec{B}_{aE}$,式中 $\vec{B}_{aE} = g_{a\gamma\gamma}\frac{\mu_r}{c}(a\vec{E})$

c 是光速且

$$c = \sqrt{\frac{1}{\varepsilon_0\mu_0}}$$

对上述方程的修改($\vec{E}, \vec{B}$)

利用: $\vec{E}_T = \vec{E} + \vec{E}_{aB}$,式中 $\vec{E}_{aB} = -g_{a\gamma\gamma}\frac{c}{\varepsilon_r}(a\vec{B})$

$\vec{B}_T = \vec{B} + \vec{B}_{aE}$,式中 $\vec{B}_{aE} = g_{a\gamma\gamma}\frac{\mu_r}{c}(a\vec{E})$

消去$\vec{E}_{\mathrm{aB}}$和$\vec{B}_{\mathrm{aE}}$,得:

$$\begin{aligned}\vec{E}_T &= \vec{E} - g_{a\gamma\gamma}\frac{c}{\varepsilon_r}(a\vec{B}_T) + a g_{a\gamma\gamma}^2 \frac{c}{\varepsilon_r}(a\vec{E}) \\ &\approx \vec{E} - g_{a\gamma\gamma}\frac{c}{\varepsilon_r}(a\vec{B}_T)\end{aligned}$$

$$\begin{aligned}\vec{B}_T &= \vec{E} + g_{a\gamma\gamma}\frac{\mu_r}{c}(a\vec{E}_T) + a g_{a\gamma\gamma}^2 \frac{\mu_r}{c}(a\vec{B}) \\ &\approx \vec{B} - g_{a\gamma\gamma}\frac{\mu_r}{c}(a\vec{E}_T)\end{aligned}$$

结合前述公式，可得:

修改后的方程组

$$\nabla \times \vec{E}_T + \frac{\partial \vec{B}_T}{\partial t} = -g_{a\gamma\gamma} a \frac{c}{\varepsilon_r} \mu_r \mu_0 \vec{j}_0$$

$$\nabla \cdot \vec{E}_T = \frac{\rho_0}{\varepsilon_r \varepsilon_0}$$

$$\nabla \times \vec{B}_T - \frac{\varepsilon_r \mu_r}{c^2} \frac{\partial \vec{E}_T}{\partial t} = \mu_r \mu_0 \vec{j}_0$$

$$\nabla \cdot \vec{B}_T = -g_{a\gamma\gamma} a \frac{c}{\varepsilon_r} \mu_r \mu_0 \rho_0$$

此时磁场成为有源场，源磁荷决定于a

一个完整的定解问题应该包含初始条件和边界条件.在稳定场中,只需要包含边界条件

*边界条件

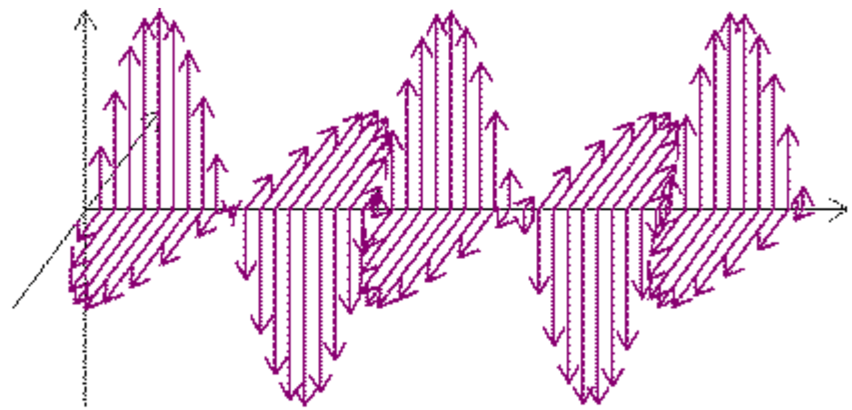
仿照电磁学中边界条件的推导,得到以下边界条件:

$$\vec{E}^{\perp}_{T1} = \vec{E}^{\perp}_{T2}$$

$$\vec{B}^{\parallel}_{T1} - \vec{B}^{\parallel}_{T2} = \mu_0 \vec{\kappa}^{i}_{f_0} \times \hat{n}$$

$$\vec{B}^{\perp}_{T1} = \vec{B}^{\perp}_{T2}$$

$$\vec{E}^{\parallel}_{T1} - \vec{E}^{\parallel}_{T2} = -g_{a\gamma\gamma} a \frac{c}{\varepsilon_r} \mu_0 \vec{\kappa}^{i}_{f_0} \times \hat{n} = -\vec{\kappa}^{i}_{f_m} \times \hat{n}$$



*轴子激发的束缚电流和电荷

由 $\nabla \vec{D} = \rho_0 + g_{a\gamma\gamma}\sqrt{\frac{\varepsilon_0}{\mu_0}}\nabla \cdot a\vec{B}$

定义为 ρ_{aB}

由 $\nabla \times \vec{H} = \vec{j}_0 + \frac{\partial \vec{D}}{\partial t} - g_{a\gamma\gamma}\sqrt{\frac{\varepsilon_0}{\mu_0}}[\frac{\partial (a\vec{B})}{\partial t} + (\nabla \times a\vec{E})]$

$$= \vec{j}_0 + \frac{\partial \vec{D}}{\partial t} - g_{a\gamma\gamma}\sqrt{\frac{\varepsilon_0}{\mu_0}}\frac{\partial (a\vec{B})}{\partial t} - g_{a\gamma\gamma}\sqrt{\frac{\varepsilon_0}{\mu_0}}(\nabla \times a\vec{E})$$

定义为 $\vec{j}_{aB}$ 定义为 $\vec{j}_{aE}$

$$\vec{j}_{aB} = -g_{a\gamma\gamma}\sqrt{\frac{\varepsilon_0}{\mu_0}}\frac{\partial (a\vec{B})}{\partial t}$$

$\Rightarrow \nabla \cdot \vec{j}_{aB} = -\frac{\partial \rho_{aB}}{\partial t}$（连续性方程）

$$\vec{j}_{aE} = \nabla \times \vec{M}_{aE}$$

下一步计划

ADMX探测器内部

Axion Dark Matter Experiment

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9 votes, 1 answer

Q: axion couplings

As I understand it, the **axion** a originates from the spontaneous symmetry breaking of $U(1)_{PQ}$. This symmetry being anomalous, and because of the QCD vacuum structure, a non vanishing term like ... $\frac{a}{f_a}Tr(G\tilde{G})$ is included in the Lagrangian, where G is the gluon field strenght. This determines the **axion** couplings to gluons. Talking about a coupling to photons would mean to ...

quantum-field-theory · cp-violation · beyond-the-standard-model

asked Jun 7 '14 by user42865

2 votes, 1 answer

Q: Axion related questions

I have several question regarding **axion**. Could anyone give me some brief introduction to what is a **axion** string, **axion** field and how is this related to fermion zero mode and chiral zero mode? ...

quantum-field-theory · condensed-matter · quantum-chromodynamics

asked Mar 6 '11 by huyichen

1 vote, 1 answer

Q: Axion coupling to photons

I dont understand the answer to this question: **axion** couplings There are experiments trying to measure how light is shining through a wall, using the coupling of the strong cp fixing **axion** to the sm ... photon. But is this coupling of the **axion** directly to the photon (as I saw in many related Feynman diagrams) or via a fermion loop as suggested by the answer in the above stated link? So in other ...

beyond-the-standard-model

asked Mar 29 '17 by Mr Puh

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Physics Stack Exchange

Q:

How do we see that the axion is a pseudoscalar?

Asked 11 months ago   Active 11 months ago   Viewed 251 times

A:

1 Answer

active   oldest   votes

1

If a were a scalar you could not have generated that term in the first place.

OK, I would defer to Peccei's 2006 review of which this is essentially an annotation. I'll be cavalier about precise normalizations and such, and use lots of ~ s and hand waving analogy, since the issues are conceptual and strategic, not hard calculations relying on impossible strong coupling QCD answers.

Let me first review the SSB of the standard axial $J_\mu^{5\,3}$ in low energy chiral symmetry breaking in QCD. To simplify things, ignore the charged pions and isospin, so π means $\pi^0 = \pi^3$, to focus on the heart of the process, chiral symmetry in the σ-model, through a mock axial U(1), really cartooning the three axials flanking the isospin SU(2) vector currents:

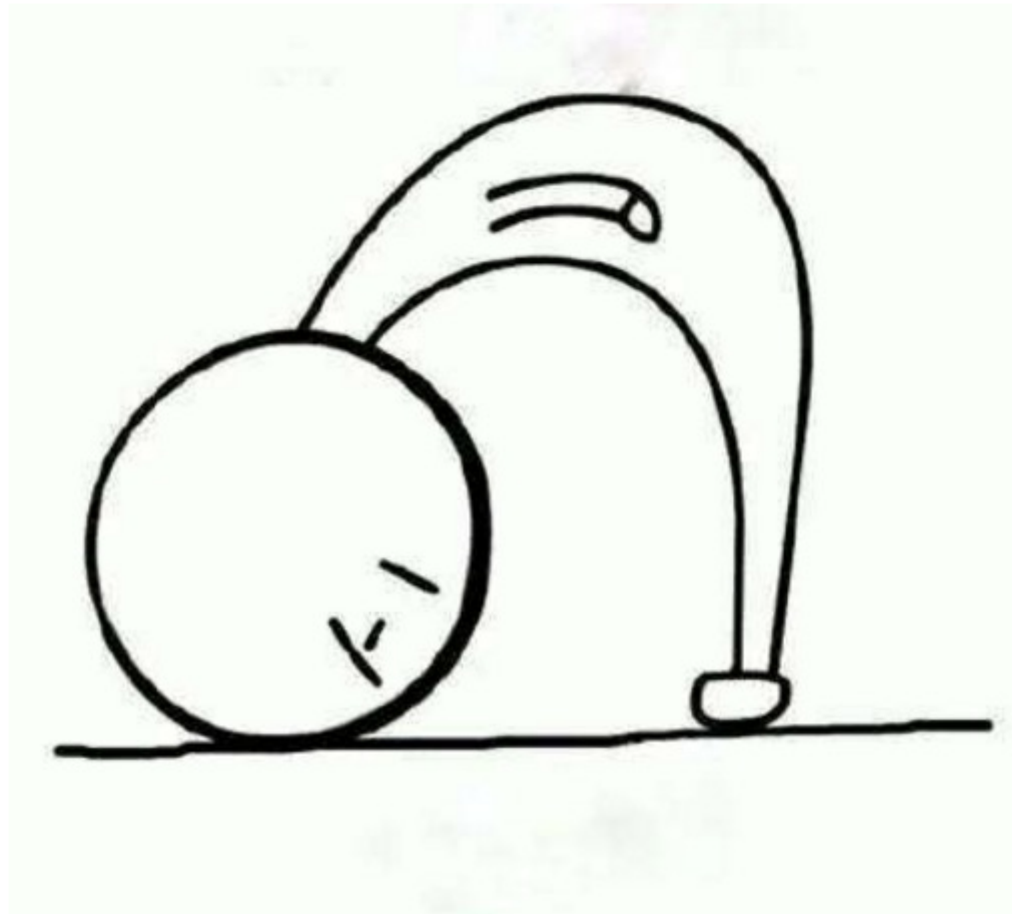
$$\delta\sigma = \pi, \qquad \delta\pi = -\sigma.$$

QCD interactions can be summarized by a mock effective potential, $\lambda((\pi^2 + \sigma^2) - f_\pi^2)^2$, whose minimum dictates that, at the ground state $\langle\sigma\rangle = f_\pi$; so we redefine $\sigma = \sigma' + f_\pi$, so that the v.e.v.s of both π and σ' are now at 0, as they should be.

You can work out the σ' has a mass, but not the π, as required by the Goldstone theorem. The goldston is always the particle that rotates to a constant,

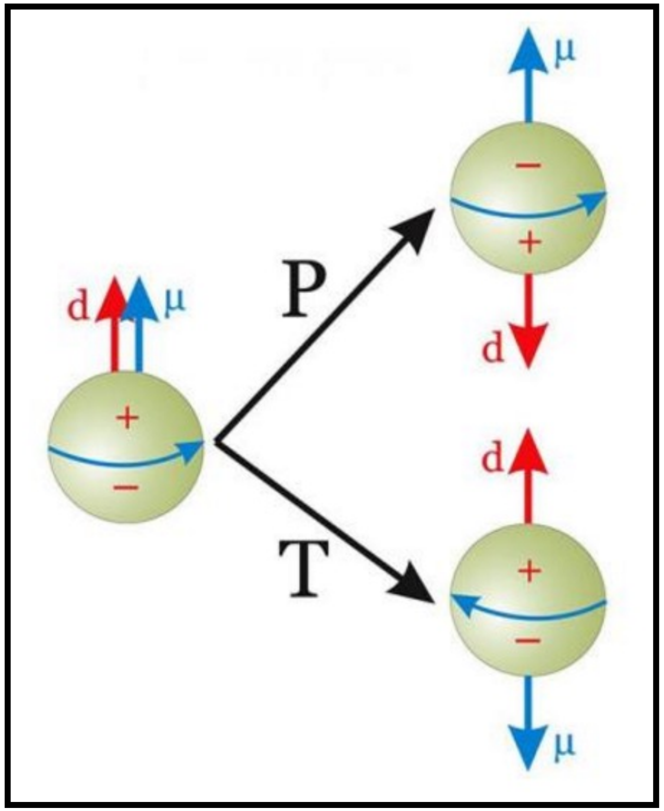
$$\langle\delta\sigma'\rangle = \langle\pi\rangle = 0, \qquad \langle\delta\pi\rangle = -\langle\sigma\rangle = -f,$$

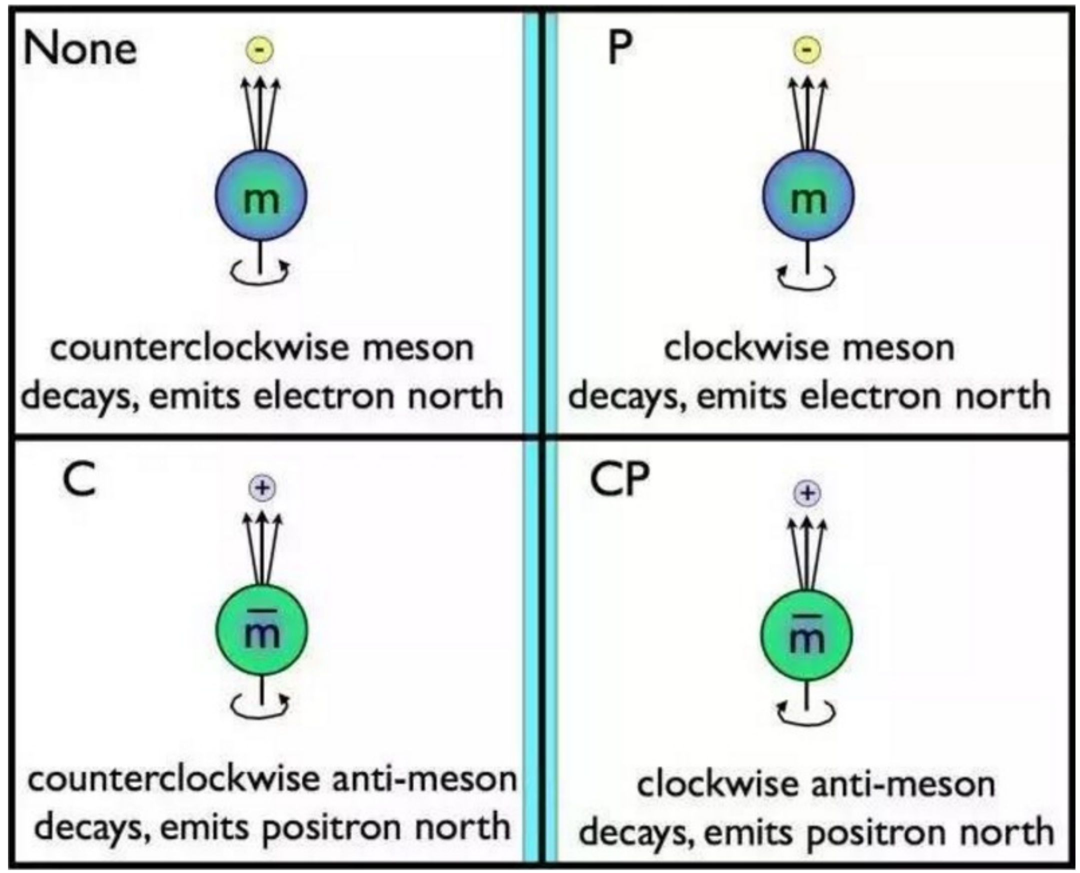
谢谢！



强相互作用中允许出现CP对称性破缺，但这种破缺至今没有被检测到。

于是出现两种可能，一是测量技术有限，所以未能检测到；二是存在新的物理规律使得CP对称性破缺被禁止。在这种新的物理规律下，就有了轴子。





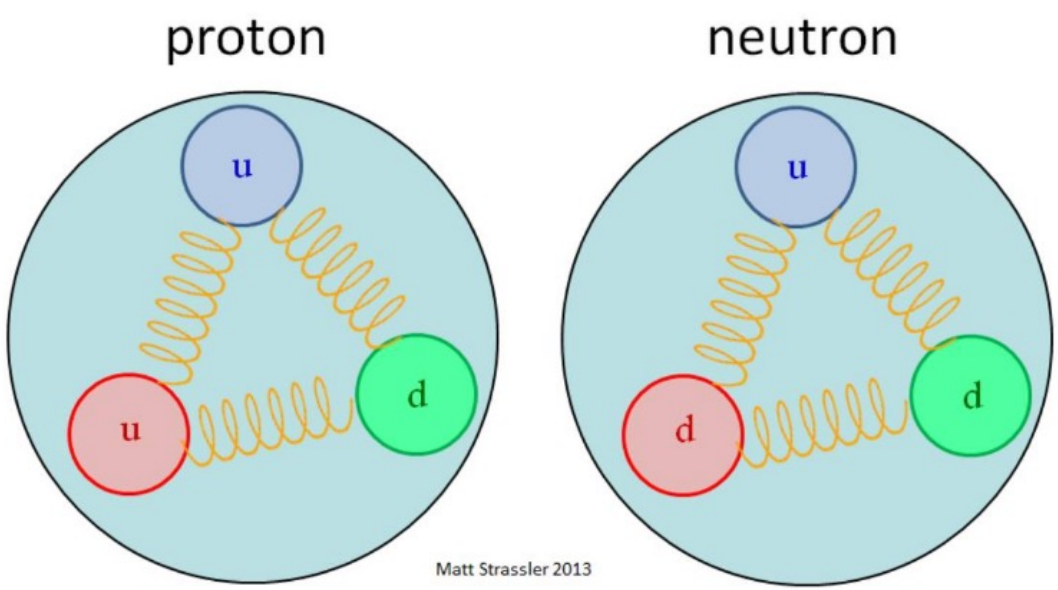
强CP问题（下）

具体而言，以Peccei-Quinn的理论为例。

写出QCD中的拉格朗日密度：

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu - m e^{i\theta'\gamma_5})\psi - \frac{1}{4}\,\mathrm{tr}\ F_{\mu\nu}F^{\mu\nu} - n_f\frac{g^2\theta}{32\pi^2}\,\mathrm{tr}\ F_{\mu\nu}\tilde{F}^{\mu\nu}$$

他们认为参数θ并不是常数，而是一个随时间演化的场。在量子场论中，一个场对应着一个粒子，这里对应的粒子就是轴子。



Campton效应

X光与电子散射时波长会发生移动，即散射光中除了有原波长L_0的散射光外，还出现了波长$L > L_0$的X光。波长的增量随散射角的不同而变化，这种现象称为Campton效应

波长偏移公式：$\Delta\lambda = \lambda - \lambda_0 = \frac{2\text{h}}{\text{mc}}\sin^2\frac{\theta}{2}$

电子的Campton波长：$\lambda = \frac{\text{h}}{mc}$