(Journal Club)

Reconstructing 3D structure of microbunched electrons from plasma wakefield based on coherent optical transition radiation[1]

Ze Ouyang

Feb 29th, 2024

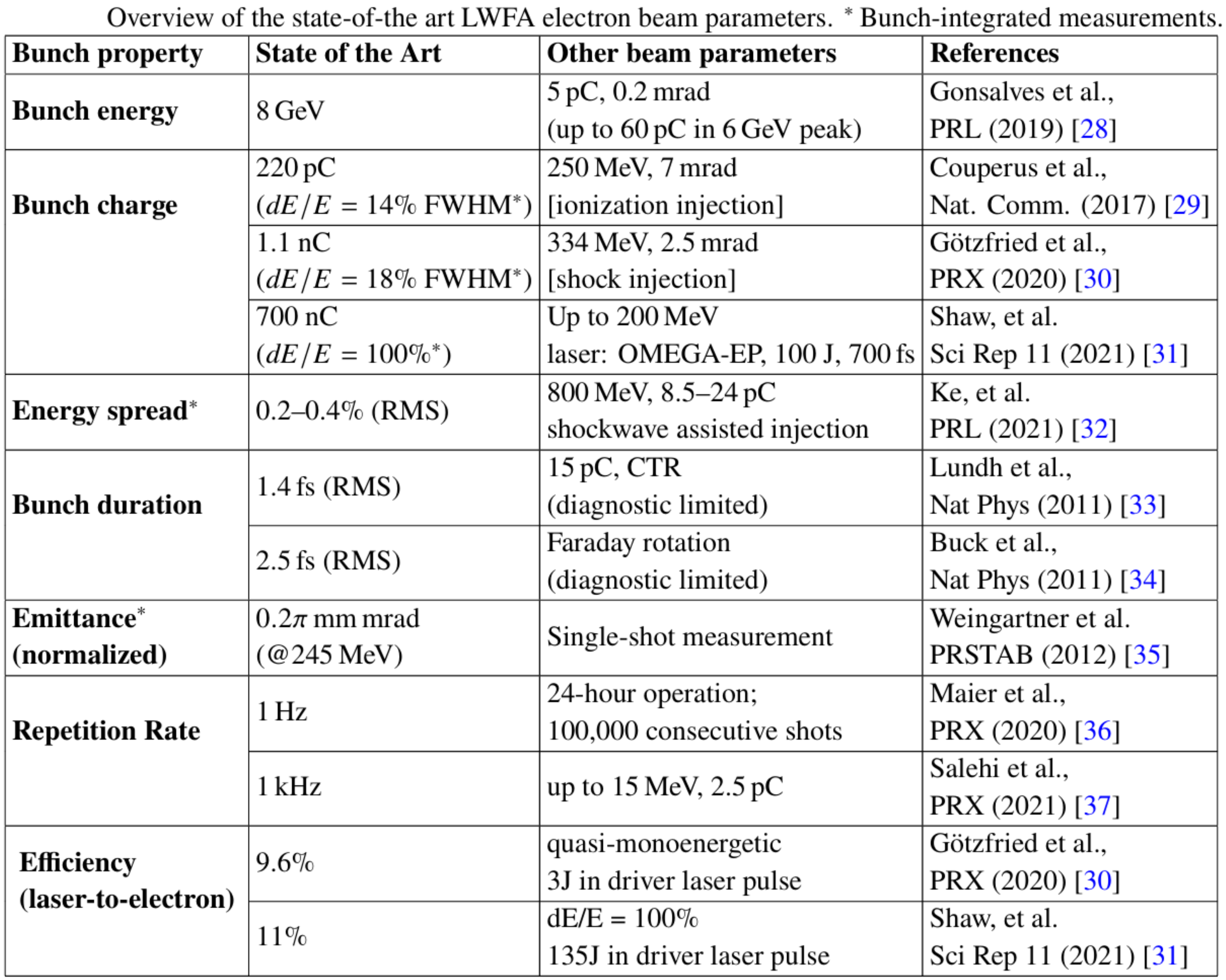
---

[1] https://www.researchsquare.com/article/rs-3894996/v1

Content

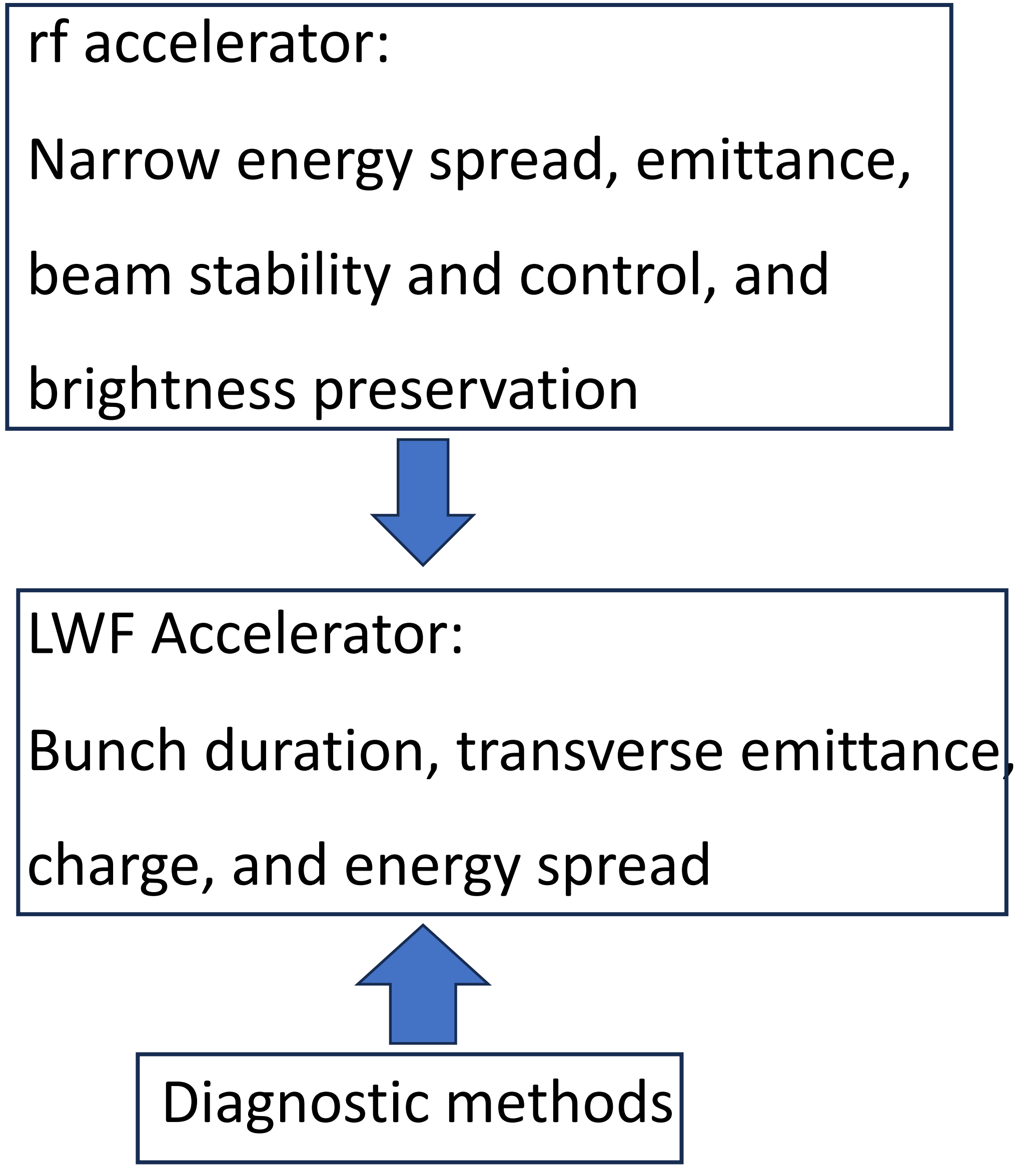
1. Introduction & Concepts
2. Experiments: injection, and COTR generation & imaging
3. Reconstruction
4. Conclusion

Introduction



Overview of the state-of-the art LWFA electron beam parameters. * Bunch-integrated measurements.

Bunch property	State of the Art	Other beam parameters	References
Bunch energy	8 GeV	5 pC, 0.2 mrad (up to 60 pC in 6 GeV peak)	Gonsalves et al., PRL (2019) [28]
Bunch charge	220 pC (dE/E = 14% FWHM*)	250 MeV, 7 mrad [ionization injection]	Couperus et al., Nat. Comm. (2017) [29]
	1.1 nC (dE/E = 18% FWHM*)	334 MeV, 2.5 mrad [shock injection]	Götzfried et al., PRX (2020) [30]
	700 nC (dE/E = 100%*)	Up to 200 MeV laser: OMEGA-EP, 100 J, 700 fs	Shaw, et al. Sci Rep 11 (2021) [31]
Energy spread*	0.2–0.4% (RMS)	800 MeV, 8.5–24 pC shockwave assisted injection	Ke, et al. PRL (2021) [32]
Bunch duration	1.4 fs (RMS)	15 pC, CTR (diagnostic limited)	Lundh et al., Nat Phys (2011) [33]
	2.5 fs (RMS)	Faraday rotation (diagnostic limited)	Buck et al., Nat Phys (2011) [34]
Emittance* **(normalized)**	0.2π mm mrad (@245 MeV)	Single-shot measurement	Weingartner et al. PRSTAB (2012) [35]
Repetition Rate	1 Hz	24-hour operation; 100,000 consecutive shots	Maier et al., PRX (2020) [36]
	1 kHz	up to 15 MeV, 2.5 pC	Salehi et al., PRX (2021) [37]
Efficiency (laser-to-electron)	9.6%	quasi-monoenergetic 3J in driver laser pulse	Götzfried et al., PRX (2020) [30]
	11%	dE/E = 100% 135J in driver laser pulse	Shaw, et al. Sci Rep 11 (2021) [31]



rf accelerator:

Narrow energy spread, emittance, beam stability and control, and brightness preservation

LWF Accelerator:

Bunch duration, transverse emittance, charge, and energy spread

Diagnostic methods

1 left table courtesy of M Fuchs et al. JINST (2024)

Introduction: main content in the paper

Generating relativistic electrons from LWFA with three methods

↓

Generating multi-wavelength COTR images simultaneously

↓

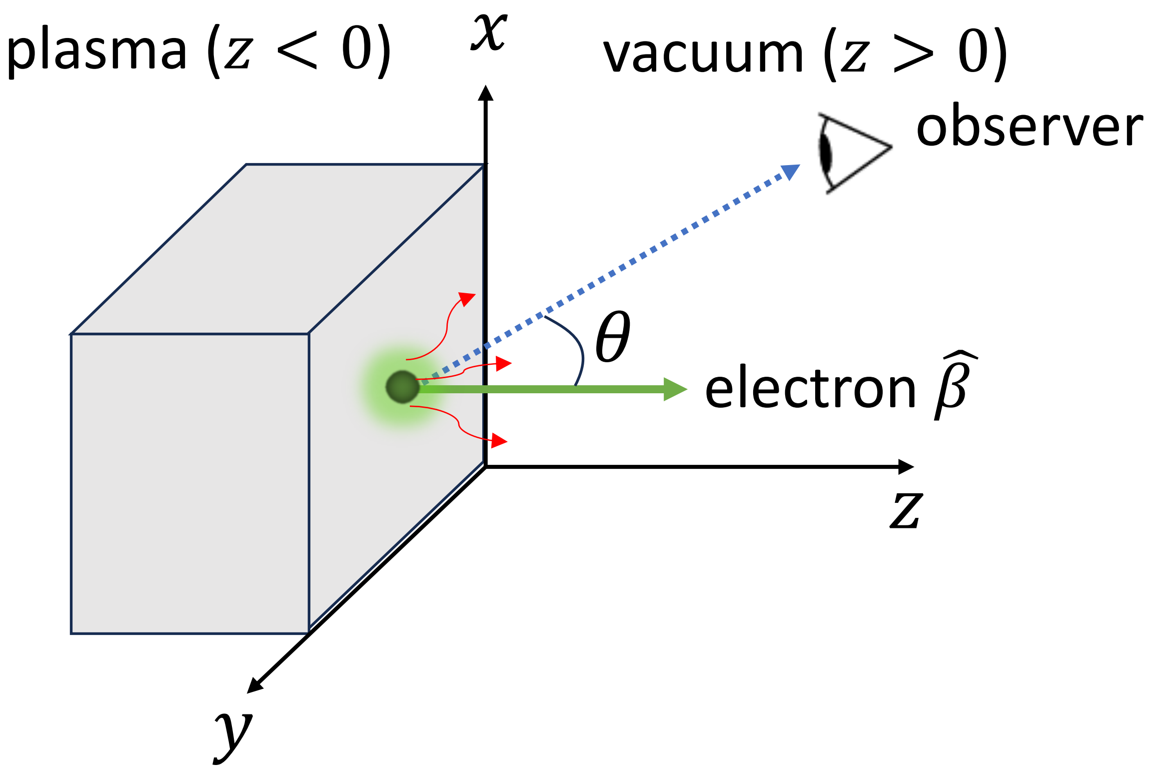
Deducing electron structures comprehensively with COTR imaging and other diagnostics

Concept: Transition Radiation (TR) of a single electron

TR is emitted when a relativistic electron passes from one medium into another with a different index of refractive.

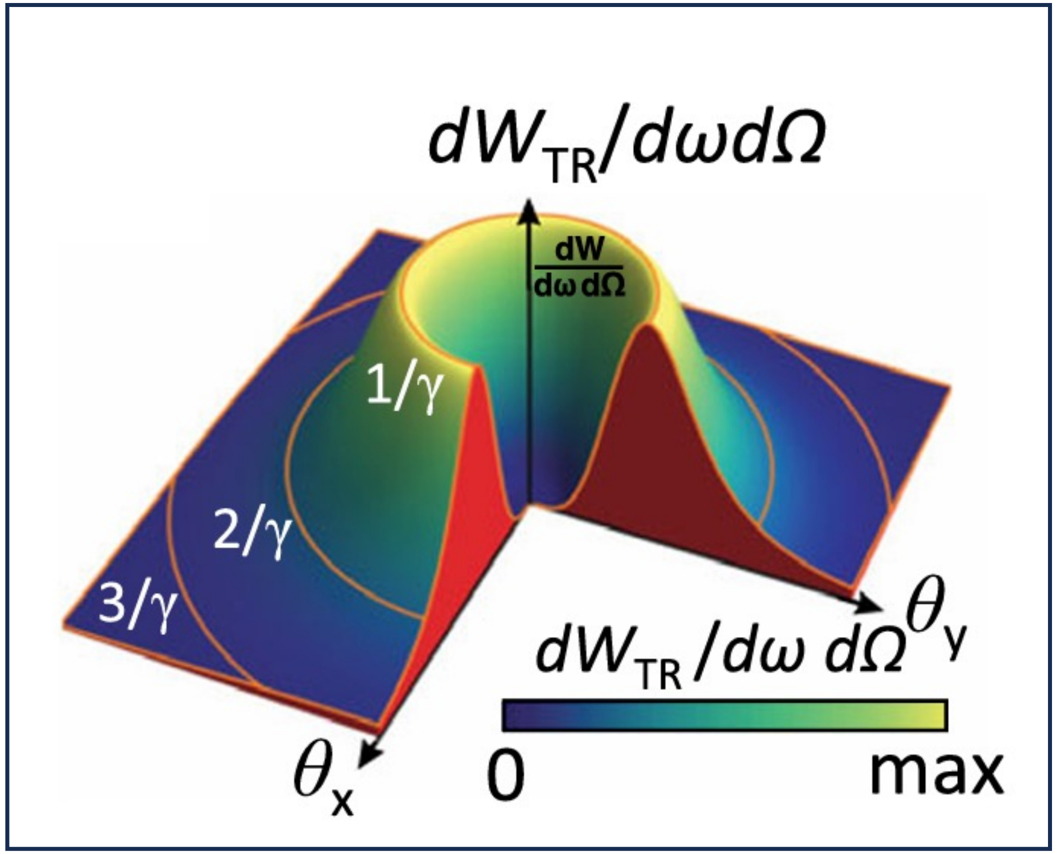
Example: Consider a single energetic electron emerging from a high density ($\omega_p^2 > \omega^2$), semi-infinite plasma normally incident into vacuum, the power distribution is given by:

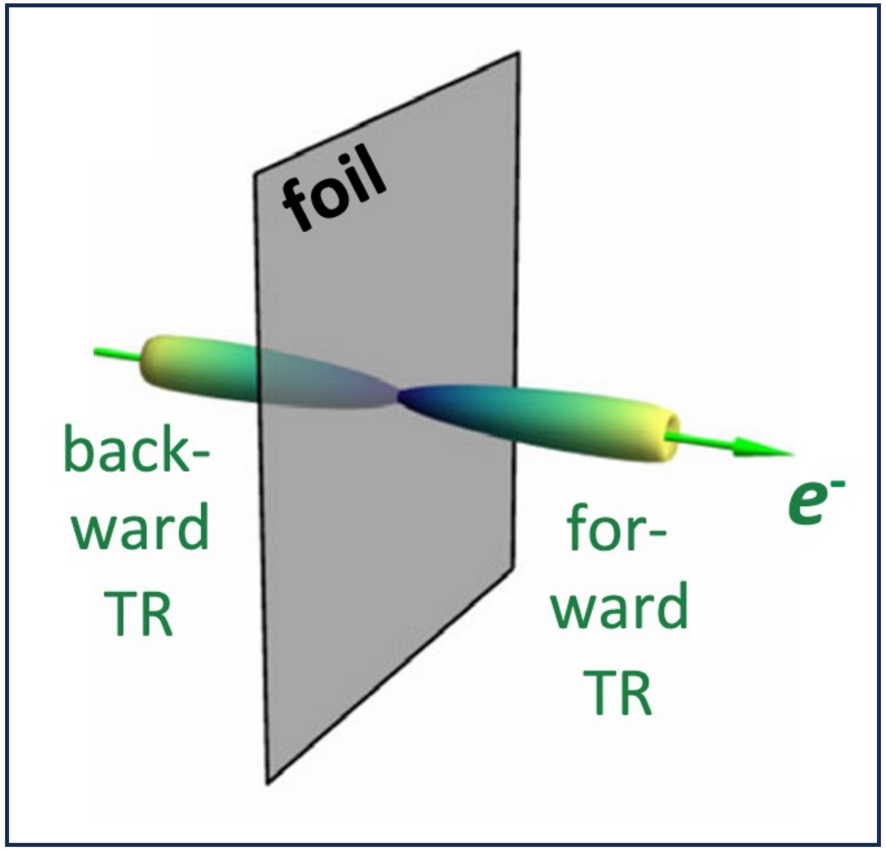
$$\frac{\mathrm{d}^2 W_e}{\mathrm{d}\omega \mathrm{d}\Omega} = \frac{r_e m_e c}{\pi^2} \frac{\beta^2 \sin^2\theta}{(1-\beta^2\cos^2\theta)^2}$$



(ω-independent)

Concept: Transition Radiation (TR) of a single electron





Unique angular distribution with

normally incident

1 pictures courtesy of M Downer et al. Rev. Mod. Phys. (2018)

Concept: Transition Radiation (TR) of multiple electrons

The TR angular distribution of multiple electrons can be written as[1] :

$$\left.\frac{\mathrm{d}^2 W_e}{\mathrm{d}\omega \mathrm{d}\Omega}\right|_{\text{multiple}} = [N_i + N_c^2 F(\omega,\theta)] \left.\frac{\mathrm{d}^2 W_e}{\mathrm{d}\omega \mathrm{d}\Omega}\right|_{\text{single}}$$

(ω-dependent)

where $F(\omega,\theta) \coloneqq F(\vec{k}) = \left|\int f(r,z) e^{(-i\vec{k}\cdot\vec{x})}\, \mathrm{d}^3 x\right|^2$ is the Fourier transform of the beam distribution(called form factor), which can either be $F(\vec{k} \coloneqq \vec{k}_\perp + \vec{k}_z)$ or $F(\vec{k}_\perp)F(\vec{k}_z)$ given certain cases. The range of the form factor $F(\vec{k})$ is $0 < \|F(\vec{k})\| < 1$.

[1] Y Liu PRL (1998)

Concept: Coherent optical transition radiation (COTR)

CTR: Typical size for the electron bunch is about several μm. The wavelength of the detectable TR should be longer than this typical size, which helps to generate Coherent Transition Radiation ($\lambda_{\mathrm{TR}} > 1\mu\mathrm{m}$).

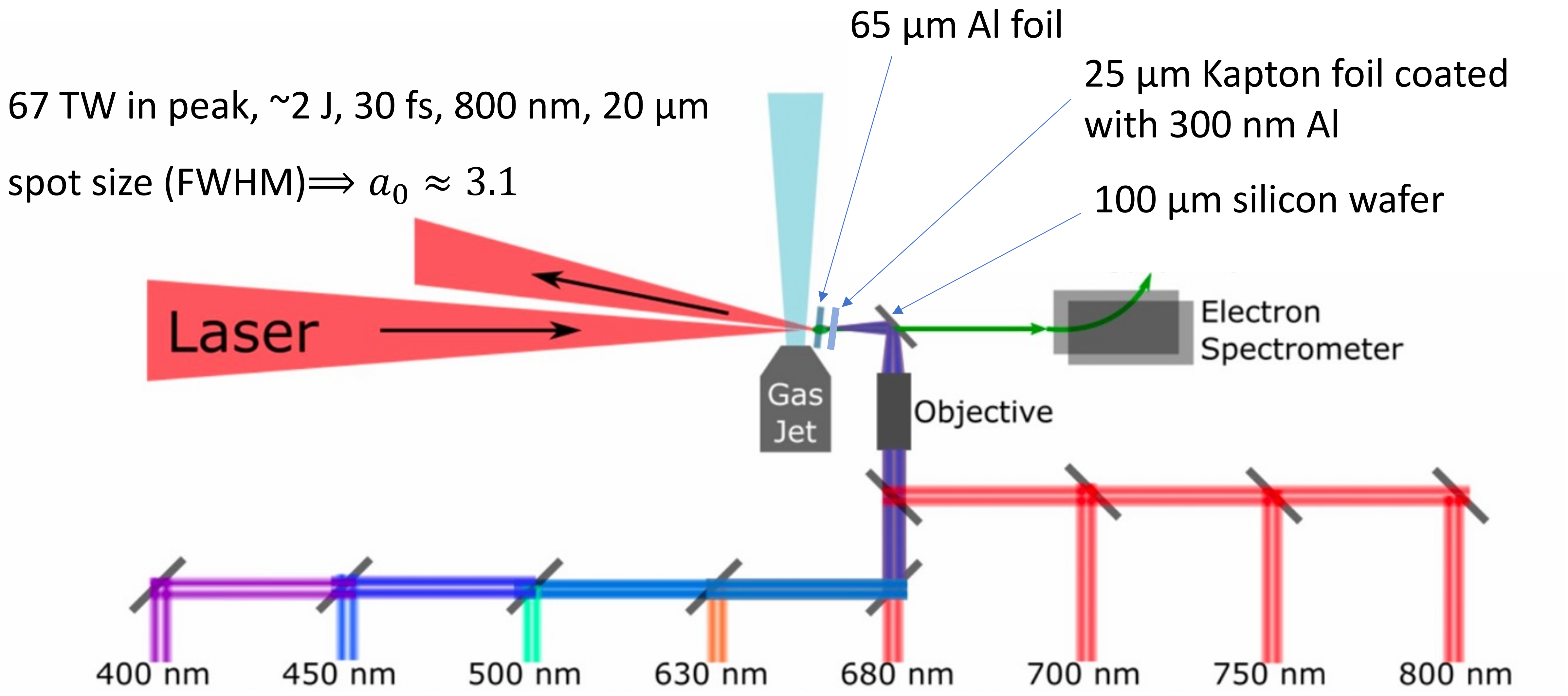
COTR: It is seen later that since the electron bunch itself got modulated to be microbunched (sub-μm), which greatly decreases its typical size. This further facilitates the generation of CTR in the optical ranges (nm), the so-called Coherent Optical Transition Radiation (COTR)[1, 2].

1 AH Lumpkin et al. PRL (2020)

2 Discussion with Ross

Experimental Setup

67 TW in peak, ~2 J, 30 fs, 800 nm, 20 µm spot size (FWHM)$\Longrightarrow a_0 \approx 3.1$



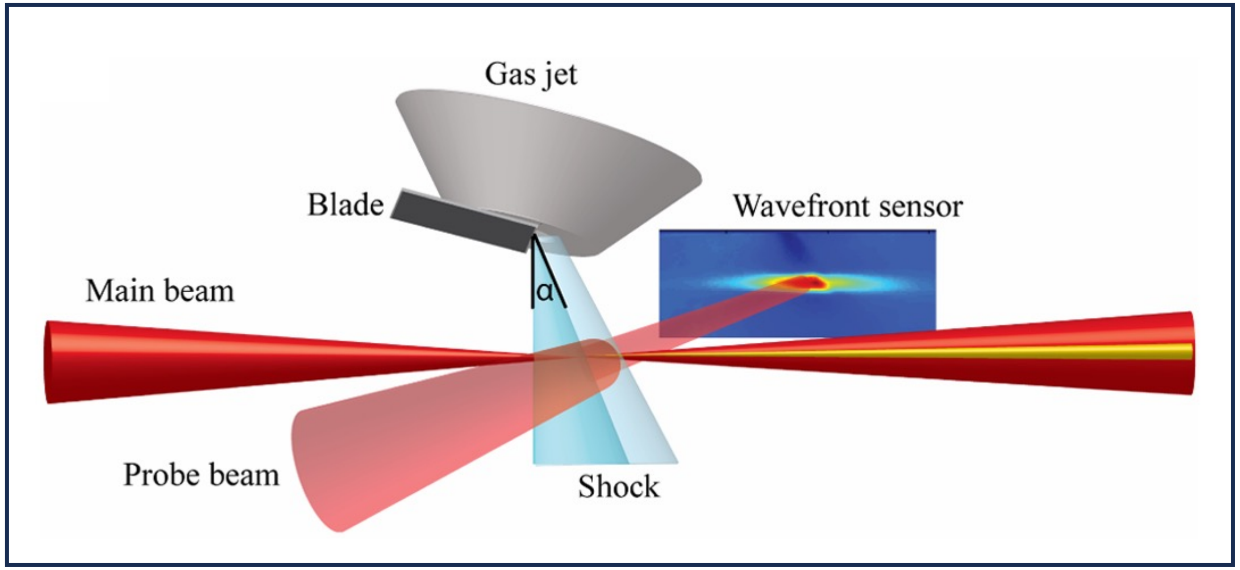
65 µm Al foil

25 µm Kapton foil coated with 300 nm Al

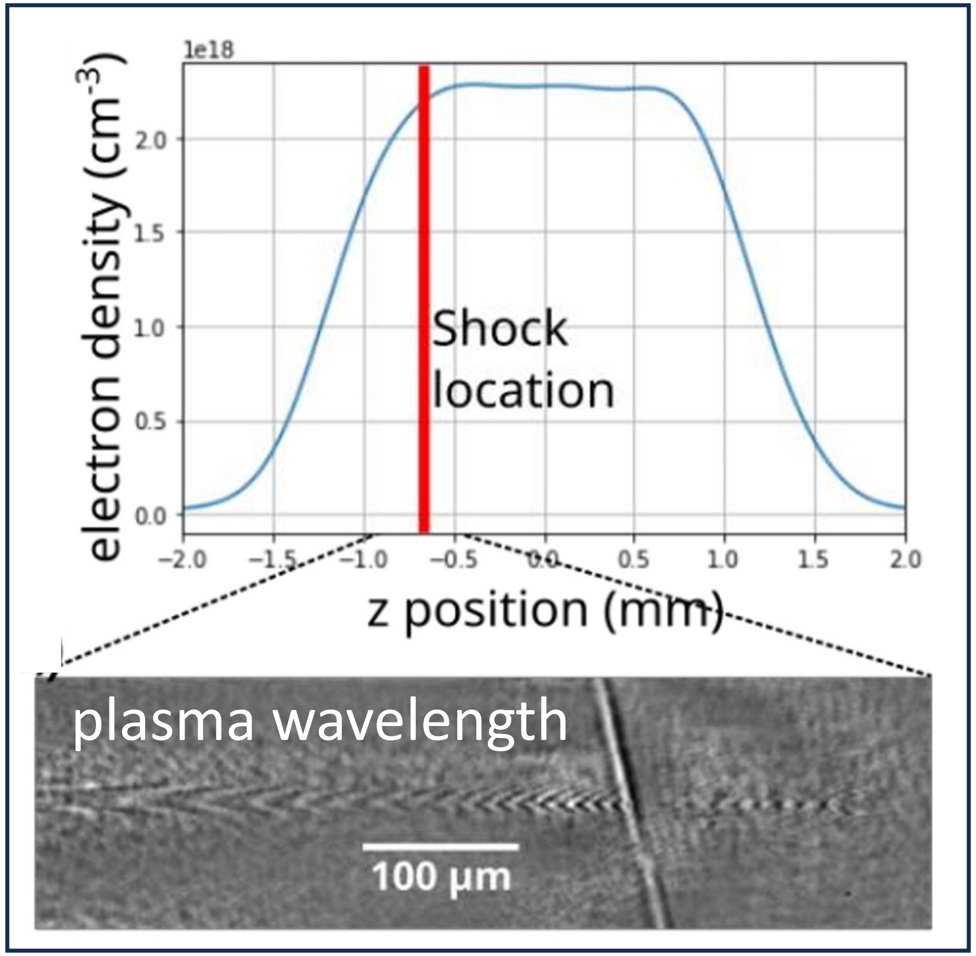
100 µm silicon wafer

Down-ramp injection

By pre-creating a sharp downward density ramp in the plasma, background electrons can be injected into the plasma wakefield at the ramp[1].



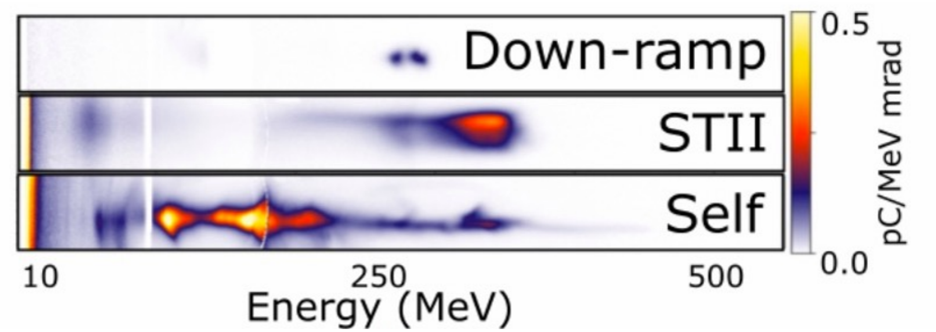
shock generation[2]



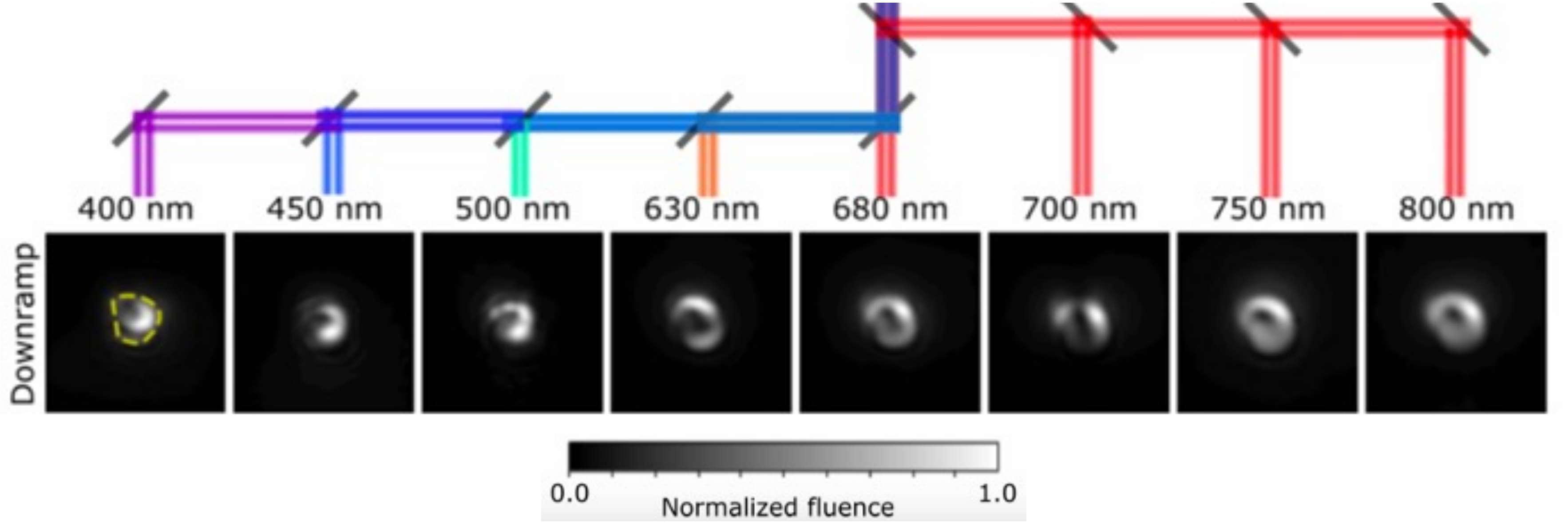
[1] TY Chien et al, PRL (2005)

[2] KK Swanson, PR Accel Beams (2017)

Results



1. quasi-monoenergetic distribution
2. total charge >100MeV is 54pC (2.3×10^8 electrons)



1. High coherence
2. Simple structure
3. Near λ-independent

Microbunched electrons $\rho(x, y, z)$ reconstruction procedure for DRI

Inverse problem: For a matrix transformation $\boldsymbol{UA} = \boldsymbol{B}$. If knowing $\boldsymbol{B}$, how to get $\boldsymbol{A}$?

In our case:
- $\boldsymbol{A}$: electron bunch distribution ρ
- $\boldsymbol{B}$: TR intensity profiles I
- $\boldsymbol{U}$: The mapping is $I = F(\rho)$

Universally, there are two ways to solve the ubiquitous Inverse Problems:

Way 1: Find the inverse mapping $\boldsymbol{U}^{-1}$;

Way 2: Set an initial $\boldsymbol{A_o}$ and iterate to new $\boldsymbol{A_i}$ to gradually approach the $\boldsymbol{B}$.

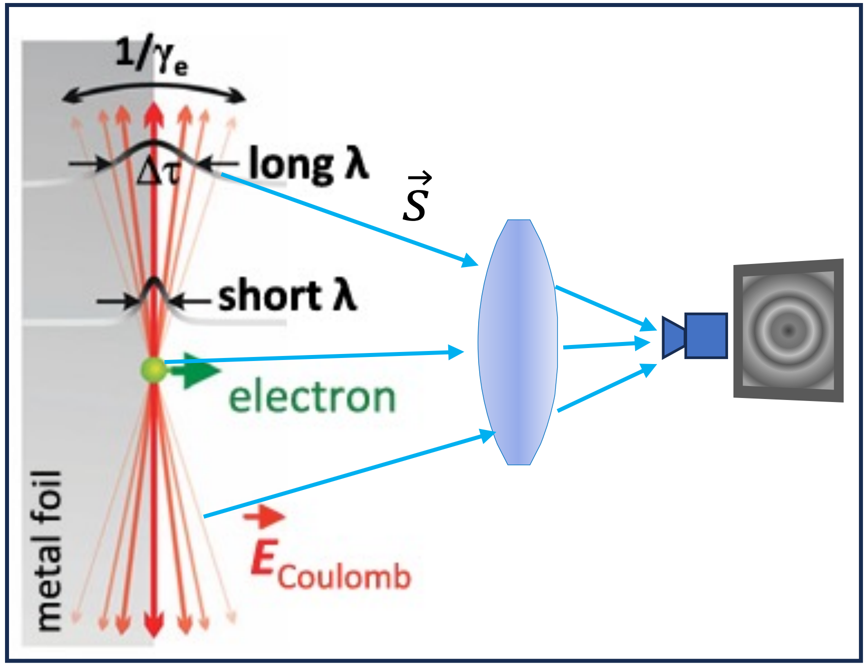
$\rho(x, y, z)$=longitudinal + transverse profile

Microbunched electrons $\rho(x, y, z)$ reconstruction procedure for DRI

Still start with a single electron. The $\vec{E}$ component of TR collected by the camera after a lens with NA=$\theta_{\max}$ is given by the Point-Spread-Function (PSF)[2]:

$$\mathbf{E}_{\perp}^{(\mathrm{PSF})}(\mathbf{r}) = \frac{2ek}{c} \int_0^{\theta_{max}} \frac{\theta^2 d\theta}{\theta^2 + \gamma^{-2}} J_1(k\theta|\mathbf{r}|)\hat{\mathbf{r}}$$

- ω-dependent
- $\vec{r}$ on the camera
- only valid for TR



For electron bunch $\rho(x, y, z)$, the field on the camera is:

$$\mathbf{E}_{\perp}^{(n)}(k, x, y) = Q \int dx' \int dy' \mathbf{E}_{\perp}^{(\mathrm{PSF})}(\mathbf{r} - \mathbf{r}') \int e^{-ikz} \rho_n(x', y', z) dz$$

Thus, the intensity detected by the camera is $\langle \vec{S} \rangle \propto \left| \vec{E}_{\perp}^{(n)}(k, x, y) \right|^2$

---

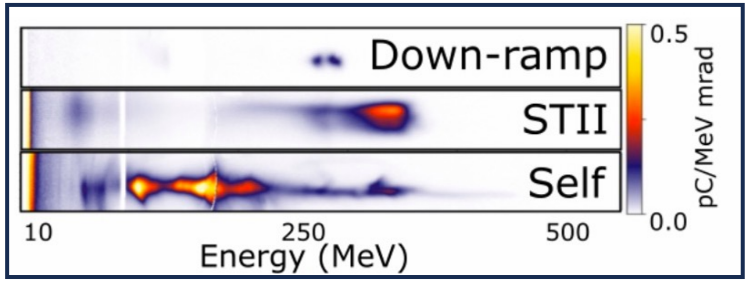
1 Picture partially from M Downer et al. Rev. Mod. Phys (2018)

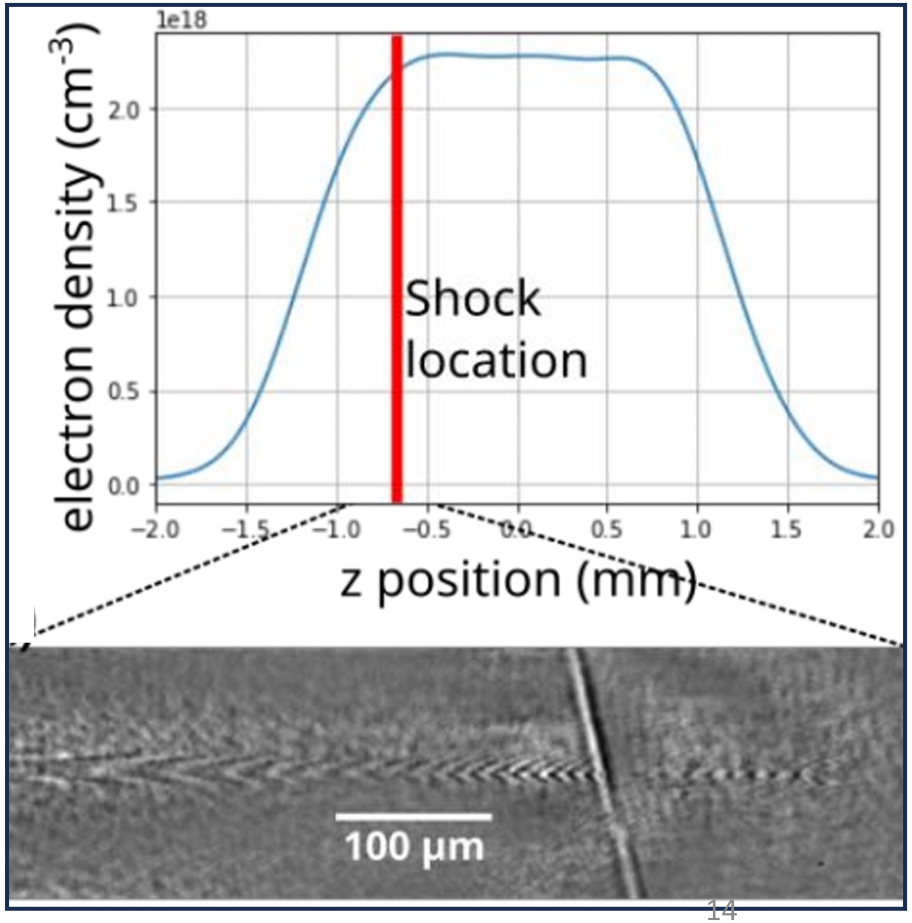
2 M Castellano et al. PR Accel Beams (1998)

Longitudinal profile reconstruction of Microbunched electrons

The bunch's longitudinal profile ($N_e(\xi)$ or $\frac{\mathrm{d}N_e}{\mathrm{d}z}$ or $\frac{\mathrm{d}N_e}{\mathrm{d}\xi}$) maps onto the bunch's energy spectrum $\frac{\partial N_e}{\partial U_e}$, if three assumptions are met:

1. Precise injection locations (confirmed by experimental data)
2. Slowly-varying wake after injection (confirmed by experimental data)
3. Chirp $\frac{\partial U_e}{\partial \xi}$ is linear or say $\vec{E}_z$ is linear with ξ (confirmed by simulation)

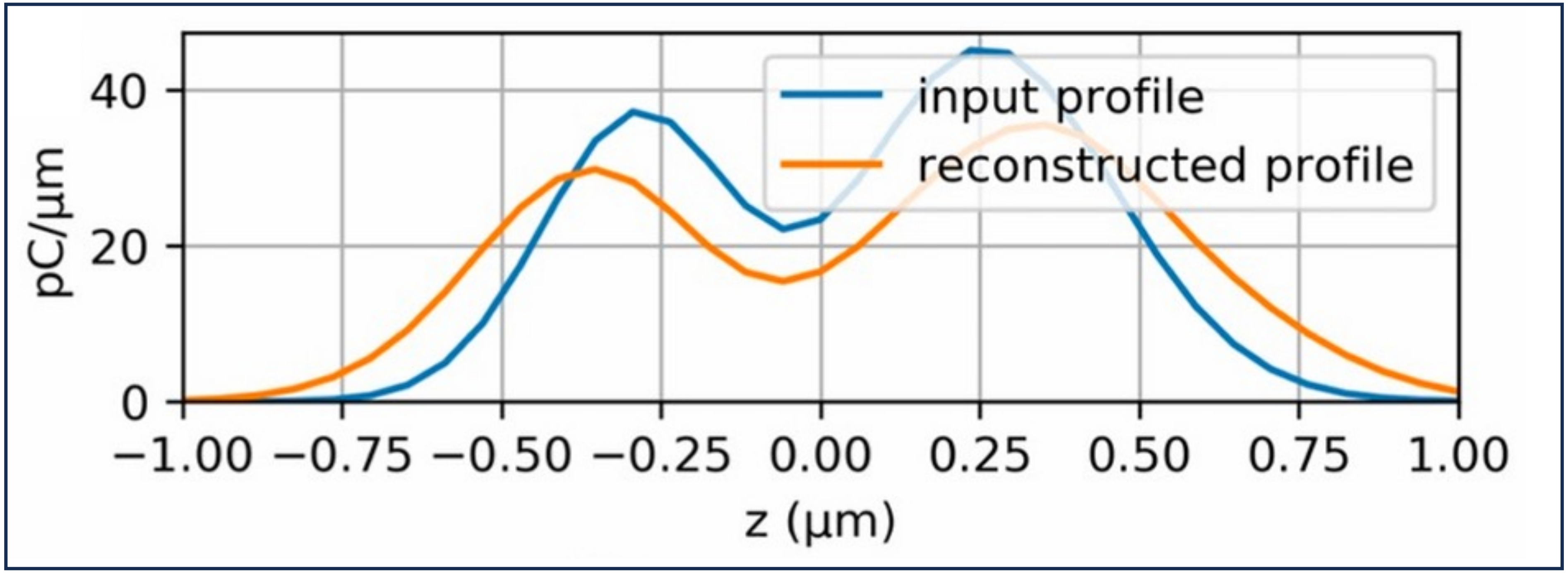




Longitudinal profile reconstruction of Microbunched electrons

Given $E_z = \frac{en_e}{2\epsilon_0}\xi$, then the potential is $\Phi = \frac{en_e}{4\epsilon_0}\xi^2$, and the energy gain is $U_e = e(\Phi_i - \Phi_f)$.

The longitudinal profile is $\frac{\mathrm{d}N_e}{\mathrm{d}\xi} = \frac{\partial N_e}{\partial U_e}\frac{\partial U_e}{\partial \xi} = \frac{\partial N_e}{\partial U_e}\frac{n_e e^2}{2\epsilon_0}(\xi_i - \xi_f) = \frac{n_e e^2}{2\epsilon_0}\frac{\partial N_e}{\partial U_e}L_{\mathrm{acc}}$



(Just exited the plasma)

Transverse profile reconstruction of Microbunched electrons

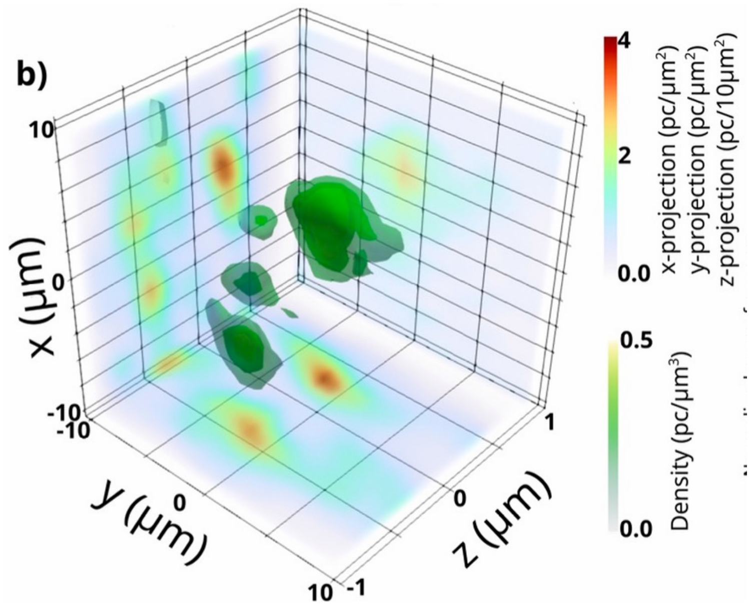
Randomly set the initial $\rho_0(x, y, z)$ ⇒ Generate COTR

A cost function NSS(Normalized Sum-of-Squares) is used to guide the new $\rho_i(x, y, z)$, based on the estimated longitudinal profile, measured total charge, and matching degree to the measured multi-spectral COTR.

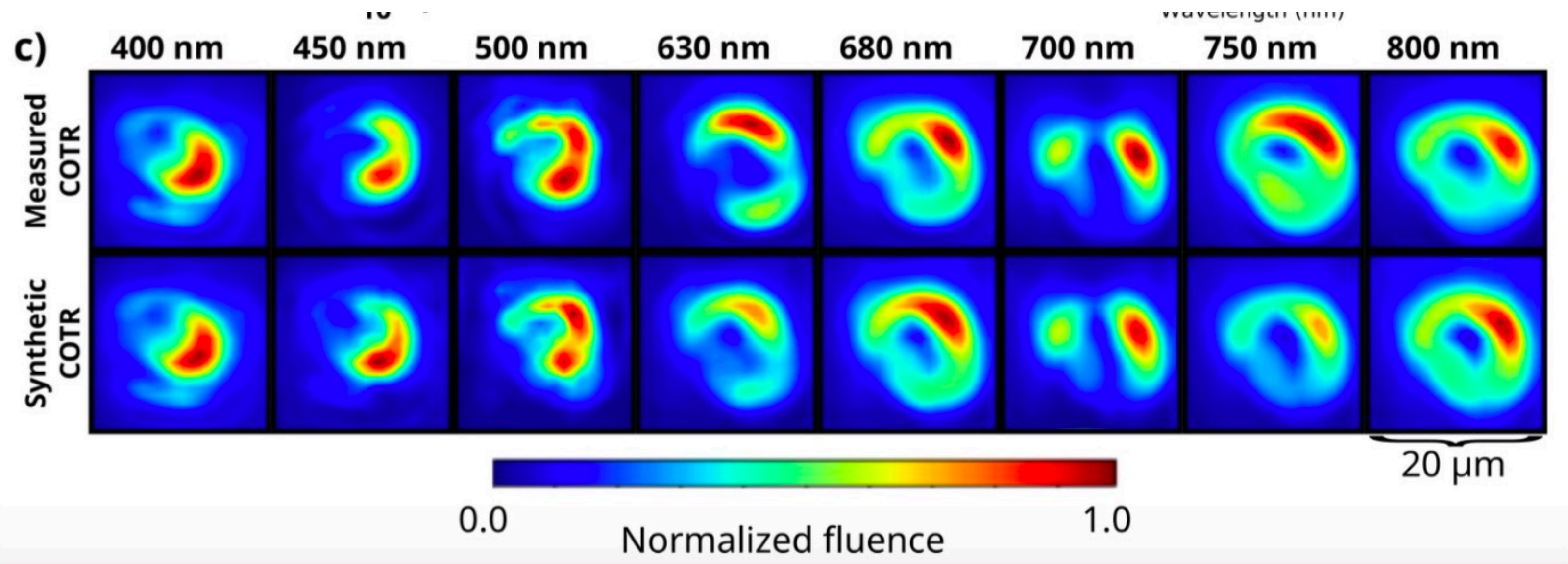
Replace $\rho_0(x, y, z)$ to $\rho_i(x, y, z)$ ⇒ Generate COTR ⇒ Guided by the cost function again

Converge to final $\rho_f(x, y, z)$

Profile analysis (bunch shape)



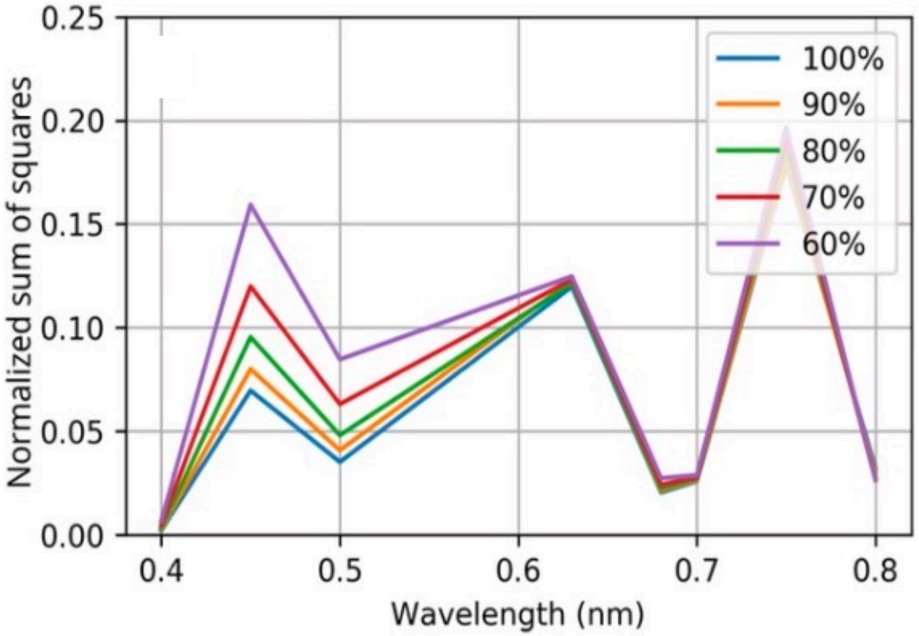
1. Transverse coherence is low
2. Longitudinal microbunched electrons found
3. Several typical converged solution sets exist



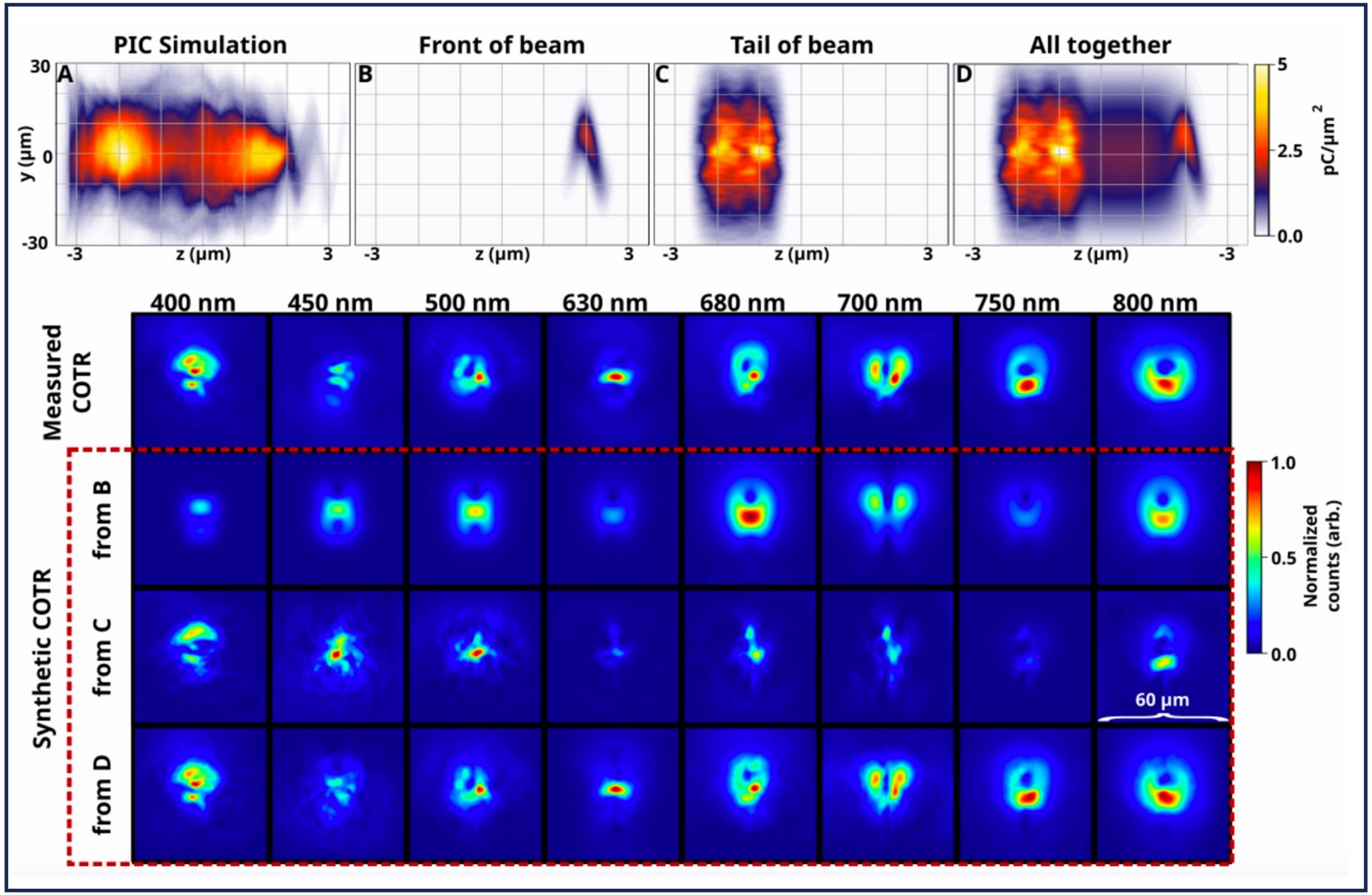
Profile analysis (form factor)

$$\left.\frac{\mathrm{d}^2 W_e}{\mathrm{d}\omega \mathrm{d}\Omega}\right|_{\text{multiple}} = [N_i + N_c^2 F(\omega,\theta)] \left.\frac{\mathrm{d}^2 W_e}{\mathrm{d}\omega \mathrm{d}\Omega}\right|_{\text{single}} \Rightarrow \frac{W_{\text{camera}}(\lambda)}{W_{\text{single}}(\lambda)} = N_i + N_c^2 F(\omega,\theta)$$

Assuming $F(\omega,\theta)$=1, we can set the lower limit of $\frac{N_c}{N_e}\sim 0.05$. By decreasing the charge slightly, the $\frac{W(\lambda)}{W_{\text{single}}(\lambda)}$ drops significantly (indicated by the NSS vs. charge), which means the N_c dominates. Thus, $\frac{N_c}{N_e}\sim 1$. This further indicates the $F(\omega,\theta)$ is $\ll 1$.

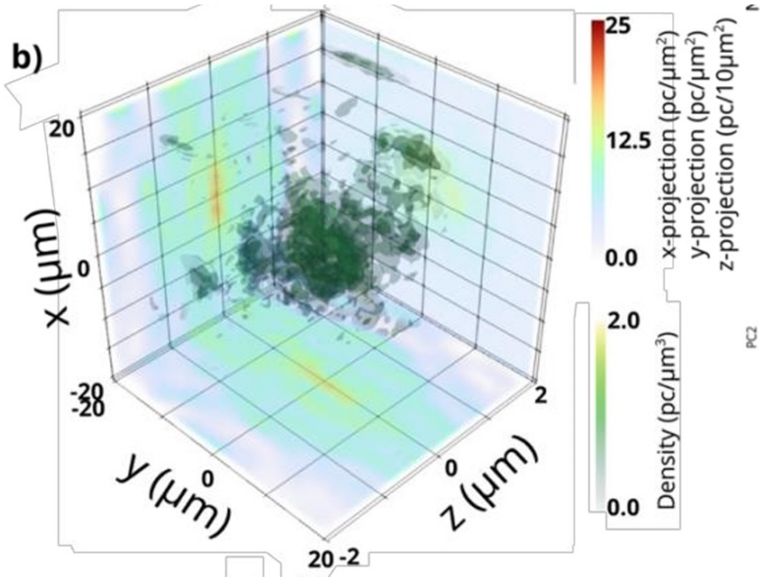


Self-truncated ionization injection

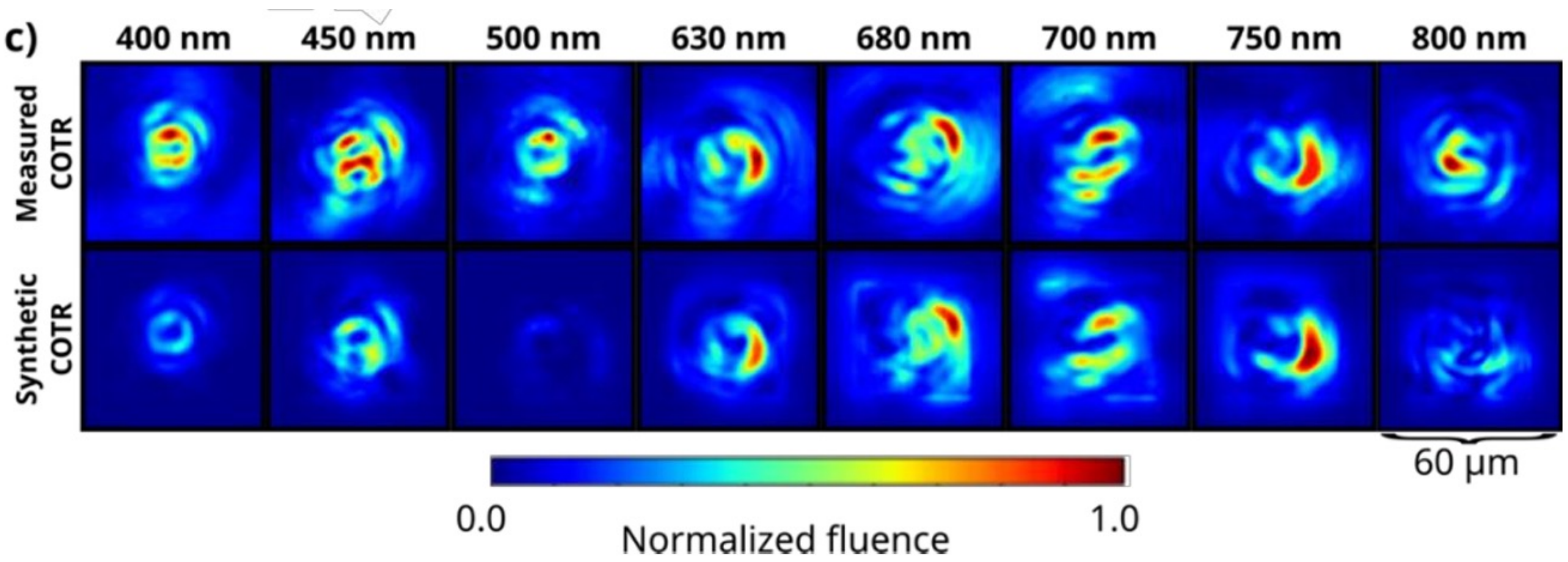


1. ~300 pc (1.2×10^9 electrons)
2. Instead of the electron spectrometer, the longitudinal profile is from PIC simulation
3. Portion contribution

Self injection



1. Lack of pre-known longitudinal profile, not-satisfying simulation result, wide transverse region
2. Time-consuming reconstruction based Gaussian longitudinal profile
3. Various solutions exist



Conclusion

1. Demonstrated multispectral COTR imaging as a high-resolution diagnostic of the microbunched electrons.
2. The microbunched electrons are required with known longitudinal profile
3. Works well in Down-ramp injection and Self-truncated ionization injection, but needs to be improved in terms of the Self-injection.
4. A promising method to improve LWFA-driven X-ray sources [1], because of the contribution of coherent electrons in X-ray also being N_c^2.

[1] S Corde et al *Rev. Mod. Phys* (2013)

Thanks!